POLYTHROPIC MODEL OF THE COMPONENT OF CLOSE BINARY SYSTEM

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ABSTRACT. In the present work we tried to resolve the problem about the detailed interior structure of the non-single star which is the component of close binary system. We have consider the case of self-gravitation potential of the gaseous sphere, which is influenced by the forces connected with rotation and tidal deformation from the companion.

Considered models appear to be remarkably centrally condensed when compared with corresponding models of the non-rotating single stars. This circumstance can lead to some evolutionary changes in the close binary systems, where one or two components

Key words: Stars: Binaries: Roche lobe, Inner Structure

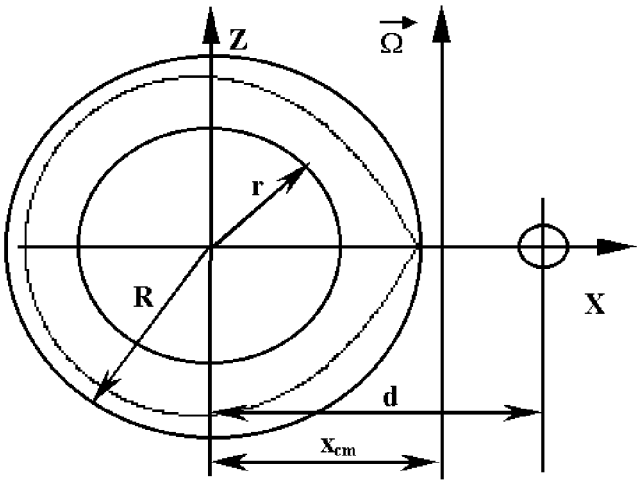

Figure 1: Graphical representation of the considered model

Introduction

For a long time, so-called Roche lobe model was used for description of the centrally condensed synchronously rotating components of the close binary systems. Accordingly to such a model, each component is presented by the point mass surrounded by the diluted shell with negligibly small density. Within this model one can obtain an exact expression for surface levels of a binary star.

In the present work we tried to resolve the problem about the detailed interior structure of the non-single star which is the component of close binary system . For this aim we used self consistent field method (SCF), first employed by Ostriker et al., (1968) for investigation of the interior structure of the rotating stars (Ostriker and Mark, 1968).

The stellar structure depends upon various parameters. To simplify the problem, we adopted the following assumptions:

1) secondary component is a point mass having the corresponding gravitational potential;

2) angular rotational velocity of the primary equals to its orbital velocity (i.e., rotation and orbital movement are synchronous);

3) density distribution in primary is given by the barotropic equation;

4) the influence of magnetic fields and external forces is negligible.

Masses of primary and secondary components are M_1 and M_2, the distance between the centres of mass of the components is d (fig.1), angular velocity of rotation is ω. The beginning of co-ordinate system is set in the centre of mass of primary component, x-axis is directed towards the centre of mass of the secondary and z-axis is parallel to $\vec{\Omega}$ vector (see Fig.1). We can write the system of equations:

$$\frac{1}{\rho}\nabla P = \nabla(U_{self} + U_{rot} + U_{sd}) = \nabla B$$
$$\nabla^2 B = -4\pi G\rho \qquad (1)$$
$$P = P(\rho)$$

where U_{self} is potential of self-gravitation of primary, U_{rot} - centrifugal potential, U_{sd} - gravitation potential of the secondary component, B - total potential, the rest definitions have usual meaning.

In case the initial distribution of the density is set, then self-gravitational potential $U_{self}(r)$ will be the sum of two values: potential formed by the mass confined within the sphere of radius r and gravity potential of the mass which is situated between the surface of the considered sphere and outher boundary of the configuration.

$$U_{self} = - G\int_0^M \mid \vec{r} - \vec{r}\,' \mid^{-1} dm' - \\ - G\int_M^M \mid \vec{r} - \vec{r}\,' \mid^{-1} dm' \qquad (2)$$

$$dm' = \rho dV' \qquad (3)$$

where M_r-mass inside the sphere of radius r, dV-elementary volume.

Obviously, the first integral corresponds to the case when $r > r'$, and second one is valid for $r < r'$. Because of the solid-state character of rotation, the centrifugal potential has the following form:

$$U_{rot} = \frac{1}{2}\omega^2((x - x_{cm})^2 + y^2) \qquad (4)$$

where x_{cm} - distance to the centre of mass of the system is given by:

$$x_{cm} = \frac{d \cdot M_2}{M_1 + M_2} \qquad (5)$$

Gravitational potential of the second component is:

$$U_{sd} = -\frac{G \cdot M_2}{\sqrt{(d-x)^2 + y^2 + z^2}} \qquad (6)$$

Having integrated the equilibrium equation

$$\frac{1}{\rho}\nabla P = \nabla B \qquad (7)$$

we get:

$$H = B - B_s, \quad H = \int \frac{dP}{\rho} \qquad (8)$$

where H -enthalpy; B_s - surface potential. In case the state equation is given, one can obtaine the following relation:

$$\rho = \rho(H) \qquad (9)$$

Relations (1)-(9) allow to calculate new density distribution. If calculated ρ_{new} differs from ρ_{old} greater than on certain value, then ρ_{new} is accepted as an initial approximation and next iteration is performed to get the self-consistence within the considered accuracy.

Computational scheme

It is reasonably to use undimensioned form of equations with the basis G,R,ρ_c, where R is a maximal radius of the star (beforehand fixed), ρ_c is a central density.

$$\rho' = \frac{\rho}{\rho_c} \\ x = \frac{r}{R} \qquad (10) \\ U' = \frac{U}{G\rho_c R^2}$$

Let us divide the spherical volume with radius R onto spherical layers having the common centre and use $\rho_{nm}(x)$ - the coefficient of density decomposition on spherical functions upon sphere of radius x [6].

$$\rho_{nm}(x) = \int_{-1}^{1} P_m^n(\mu')d\mu' \int_0^{2\pi} \rho'(x, \mu'\phi') \times \\ \times \cos(m \cdot \phi')d\phi' \qquad (11)$$

Integrals can be easily calculated using Gauss formula (ω_i,ω_k-weights, μ_k,ϕ_i - gaussian abscissae, $\mu = cos(\theta)$):

$$\rho_{nm}(x) = \sum_{k=1}^{N} \omega_k P_m^n(\mu_k) \sum_{i=1}^{N} \omega_i \rho(x, \mu_k \phi_i) \times \\ \times \cos(m \cdot \phi_i) \qquad (12)$$

Then:

$$\rho_{nm}(x,\mu,\phi) = \rho_c \sum_{n=0}^{N}\sum_{m=0}^{n} \rho_{nm}(x) P_m^n(\mu) \times \\ \times \cos(m\cdot\phi_i) \qquad (13)$$

Now, let us consider the self-gravitation potential U_{self}. On the considered sphere it consists of two components: potential Q generated by the mass from this spherical volume and potential Φ of the mass outside this sphere: $U_{self} = Q + \Phi$. Using the well-known formulae of the celestial mechanics (Duboshin, 1968), one can get the total potential as a decomposition on the spherical functions:

$$Q(x,\mu,\phi) = \sum_{n=0}^{N}\sum_{m=0}^{n} Q_{nm}(x) x^{-n-1} P_m^n(\mu) \times \\ \times \cos(m\cdot\phi_i) \qquad (14)$$

$$\Phi(x,\mu,\phi) = \sum_{n=0}^{N}\sum_{m=0}^{n} \Phi_{nm}(x) x^{n+1} P_m^n(\mu) \times \\ \times \cos(m\cdot\phi_i) \qquad (15)$$

where $\Phi_{nm}(x)$, $Q_{nm}(x)$ are:

$$Q_{nm}(x) = C_{nm} \int_0^x x'^{n+2} \rho_{nm}(x') dx' \qquad (16)$$

$$\Phi_{nm}(x) = C_{nm} \int_x^1 x'^{-n+1} \rho_{nm}(x') dx' \qquad (17)$$

$$C_{nm} = \frac{2n+1}{2\pi\delta}\frac{(n-m)!}{(n+m)!}$$
$$\delta_m = 2 \qquad m = 0$$
$$\delta_m = 1 \qquad m = 1,2,3...$$

Finally:

$$U_{self}(x,\mu,\phi) = \sum_{n=0}^{N}\sum_{m=0}^{n} U_{nm}(x) P_m^n(\mu) \times \\ \times \cos(m\cdot\phi_i) \qquad (18)$$

$$U_{nm}(x) = \Phi_{nm}(x) x^n + Q_{nm}(x) x^{-n-1} \qquad (19)$$

Calculation of the indefinite integrals (16), (17) is similar to that of the indefinite integral of the function which is tabulated. Given decompositions do not contain the sinusoidal members, under the considered co-ordinate system they appear to be zero. On each sphere with radius x, the density and potential are described as a triangle matrix:

$$\begin{vmatrix} A_{00} & 0 & 0 & 0 \\ 0 & A_{11} & 0 & 0 \\ A_{20} & 0 & A_{22} & 0 \\ 0 & A_{31} & 0 & A_{33} \end{vmatrix}$$

On the next step we should calculate the enthalpy (in the same units as potentials):

$$H = U_{self} + U_{sd} + U_{rot} - B_s \qquad (20)$$

Constant B_s from (8),(20) is determined as maximum of value B for $\mu = 0$, $\phi = 0$, $x = 1$ that corresponds to the directions towards the secondary component. Simultaneously, we keep the condition that new configuration should be situated inside the sphere $x = 1$. Stellar surface is determined by the condition $H = 0$. If we select

$$P = K\rho^{\frac{1}{n}+1} \qquad (21)$$

as a state equation, then enthalpy and density will be connected as following:

$$\rho = \left(\frac{G\rho_c R^2 H}{K(n+1)}\right)^n \qquad (22)$$

where n is a polytropic index.

Let us express K through the central density and central value of the enthalpy H_c and mass through the undimensioned mass

$$K = \frac{GR^2 H_c}{\rho_c^{\frac{1}{n}-1}(n+1)} \qquad (23)$$

$$M = \rho_c R^3 H_c^{-n} \qquad (24)$$

Undimensioned mass is:

$$M' = \int_0^1 \int_{-1}^1 \int_0^{2\pi} H^n x^2 dx d\mu d\phi \qquad (25)$$

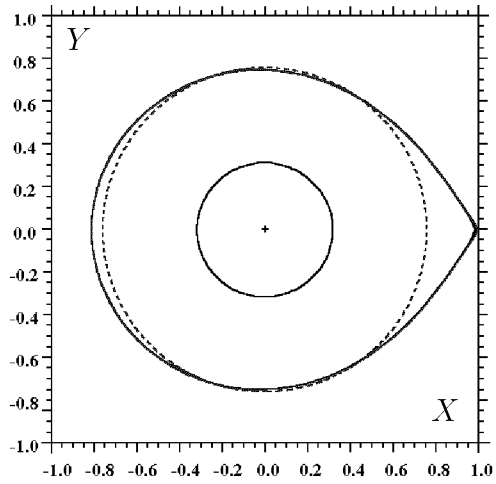

Figure 2: Equipotential surfaces. The inner one corresponds to decreasing of the potential e times with respect to its central value. $M_1/(M_1+M_2)=0.5 \quad n=3$

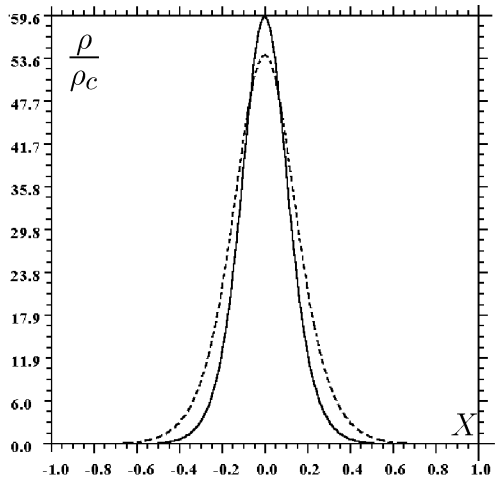

Figure 3: The density profile. $M_1/(M_1+M_2)=0.5$ $n=3$. *Dashed curve* - single star, *solid curve* - component of double system.

Let us define the undimensioned function f_h. It is Lane-Emden function in the scale where the undimensione radius of the configuration is $R=1$. Therefore, the undimensioned function f_h can be used in searching for dimensioned physical values:

$$f_h(x)=\frac{H(x)}{H_c} \tag{26}$$

The values sought for (density, pressure, etc) can be expressed using f_h function:

$$M'=\int_0^1\int_{-1}^1\int_0^{2\pi} f_h^n x^2 dx d\mu d\phi \tag{27}$$

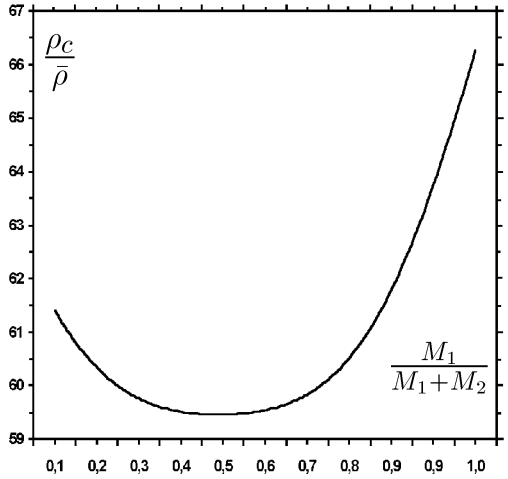

Figure 4: The dependence of ρ_c/ρ ratio upon mass ratio

$$\rho=\rho_c f_h^n \tag{28}$$

$$P=P_c f_h^{n+1} \tag{29}$$

$$K=\frac{G\rho_c}{n+1}R^{\frac{3}{n}+2}\left(\frac{M'}{M}\right)^n \tag{30}$$

Conclusion

We have investigated the problem about the interior structure of the self-gravitating gaseous sphere, which is influenced by the forces connected with rotation and tidal deformation. Obtained solution generally depends upon mass ratio $M_1/(M_1+M_2)$ and polytrope index n. It is clear, that Roche model for homogeneous incompressible ellipsoid and that for the point mass are the asymptotic solutions of the present problem (n=0 and n=3 respectively). Therefore, the configuration of the level surfaces for n=0 should well fit to the level surfaces of homogeneous ellipsoid and for $n=\inf$ to the level surfaces of point mass.

One of the most important results of the present stydy consists in the following. As one can see from fig.3, considered models are remarkably centrally condensed when compared with corresponding models of the non-rotating single stars. This circumstance can lead to some evolutionary changes in the close binary sy-

stem, where one or two components fill in their Roche lobes e.g., systems of W UMa).

Such stars can have shorter life-time depending upon the degree of additional central condensation and size of the volume inside the stellar core where it takes place. Taking into account that the rate of energy production in *pp* or CNO cycle is proportional to ρ_c^2 and T_c^n (where n varies from 4 to 20), one can conclude that even modest additional condensation of core gas can accelerate the rate of stellar evolution.

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