ANALYTICAL ESTIMATE OF INFLUENCE OF THE LIMITED ANGULAR SIZE OF THE SUN IN A PROBLEM OF TWILIGHT SOUNDING OF THE ATMOSPHERE

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ABSTRACT. The Sun has finite angular size. Therefor sunrays from different points of the solar disk have different trajectories in the atmosphere. General illumination produced by different parts of the solar disk differs even if the brightness of these parts is identical. Thus the illumination of some point in the atmosphere produced by the solar disk and the illumination of the same point produced by the Sun taken as a point source are different. It is shown that into a consideration angular size of the Sun doesn't influence the brightness of twilight sky. However, the brightness of some parts of twilight ray significantly increases. A new method of computing illumination of some point in the atmosphere is propound that takes into account angular size of the Sun.

Keywords: Earth atmosphere; aerosols; Solar radiation

1. Introduction

Except large bodies fly in the Earth's atmosphere, there are a number of small particles constantly hit it — a space dust. As dust does not radiate itself it can't be seen at night, and in daytime the scattering of Solar light caused by dust is insignificant in comparison with a scattering of solar light in the lower atmospheric layers. However, during morning and evening twilight the high atmospheric layers are irradiated much better, than lower ones. It allows defining a composition of the upper atmospheric layers by means of measuring brightness of the twilight sky.

The method of twilight sounding on measured luminosity of the twilight sky calculates a content of an aerosol in the upper atmosphere. Its procedure supposes there is a certain mode permitting to calculate a part of the twilight sky brightness that is caused by a single scattering of light on particles of an atmosphere. Theoretically, it may be found under the formula

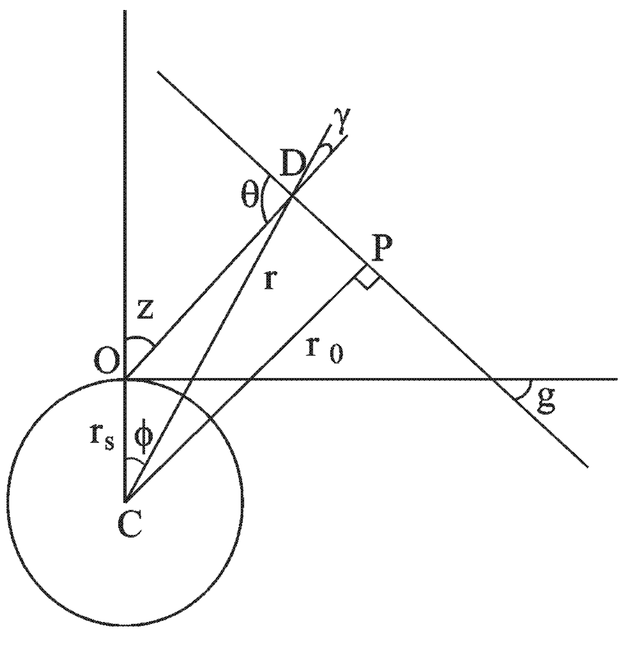

Figure 1: The plan of distribution of a not refractive ray of the Sun in Solar meridian.

$$B_1 = E_0 p^m \int_0^m \sigma(\theta, h) F(h, q) \sec\gamma dh \qquad (1)$$

Where B_1 — the brightness of primary twilight (brightness of an observable segment of the sky stipulated by single scattering);
E_0 — illumination of a site on the border of the Earth's atmosphere;
p^m — transparence of an atmosphere in a direction of a sighting;
$\sigma(\theta, h)$ — directional dissipation factor at the altitude h above a surface of the Earth;
$F(h, q)$ — transparence of an atmosphere up to a point of scattering along a trajectory of a light ray.

θ — the angle, under which scattering happens;
q — the set of parameters, from which also depends a transparence of an atmosphere up to a point of scattering.

Brightness of single scattering and transparence of an atmosphere in a direction of a sighting are being measured from observations. Illumination intensity on the border of an atmosphere is been supposed a constant. The directional dissipation factor depends on a distribution of atmospheric components, and therefore it is used for determination of a structure of an atmosphere and, hence, it is obtained from equation. The magnitude of a transparency depends on the various factors and, as a rule, calculates theoretically. For its calculation usually suppose, that the solar rays are spread rectilinearly, and the angular sizes of the Sun are neglected. Sometimes those who wish to receive results that are more precise take into account effects of a refraction and limited size of the Sun. But if effects of refraction essentially change the theoretical magnitude of brightness of single scattering, the influence of limited angular size of the Sun on magnitude of brightness of the twilight sky is estimated by authors differently. Some suppose this magnitude not essential, and others offer to take it into account. We shall try to estimate the magnitude of the correction that takes into account limited angular size of the Sun. In this time about the structure of the atmosphere, we make only common suppositions.

One of the reference suppositions of a method of twilight sounding is the supposition about a spherical symmetry of an atmosphere. It allows considering light scattering in the chosen plane. Usually it is a solar meridian. In the case, when the Sun has defined size, its part will be outside of a solar meridian, and, hence, rays will be spread outside of this meridian. Each point in the solar meridian on the disk of the Sun will correspond with a line on the disk of the Sun. However angular size of the Sun is small and radius of the Earth is large enough. Let's assume that the transparence of an atmosphere along a line on the disk of the Sun relevant to one point in a solar meridian does not vary (Link, 1962). Then the Sun can be considered as a segment in a solar meridian which length is equal to an angular size of the Sun.

The evaluation of the correction to a transparency of an atmosphere in such suppositions can be divided into two components. First, one consists in definition of a modification of illumination intensity created by a partial segment of the Sun in a solar meridian outside of an atmosphere. The second one — in the definition of a transparency of an atmosphere for each partial segment of the solar disk and evaluation of convolution of a transparency of an atmosphere and illumination intensity from each partial segment.

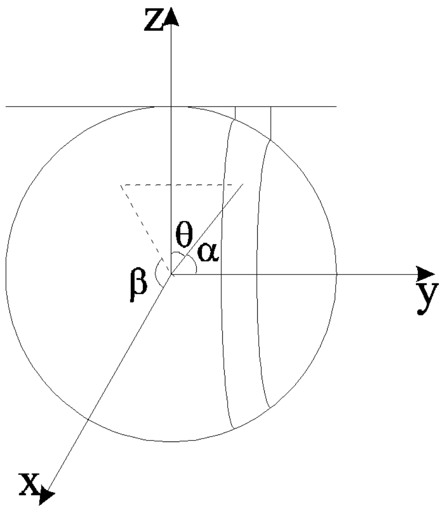

Figure 2: Coordinates on a surface of the Sun

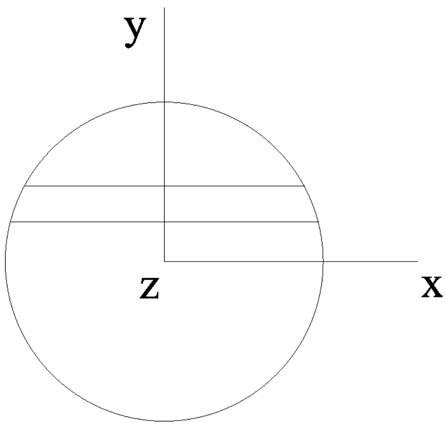

Figure 3: Aspect of a Cartesian coordinates (x, y, z) on the disk of the Sun from a place of a position of the observer.

2. Brightness of a partial segment of the Sun

The brightness of the Sun we shall name Illumination from a narrow horizontal band of the disk of the Sun referred to an angular length of a segment in a Solar meridian that is relevant to a horizontal band of the disk. Let's use spherical coordinates on a surface of the Sun. A polar angle α we'll calculate from a direction to zenith (fig. 2, 3). The z-axis is perpendicular to a picture plane. In such coordinates a band on the disk the Sun is determined by α angle. Then illumination that is created by a band of the disk of the Sun between angles α_1 and α_2 is

$$E = \left(\frac{R}{l}\right)^2 \int_{\alpha_1}^{\alpha_2} \int_0^{\pi} B(\theta) \sin^2\alpha \sin\beta d\beta d\alpha \qquad (2)$$

Where R — radius of the Sun;
l — distance from the Sun to the Earth;

$B(\theta)$ — brightness of a site on the disk of the Sun.

$$B(\theta) = B(0)(1 - u - v + u\cos(\theta) + v\cos^2(\theta)). \quad (3)$$

Where u, v - some coefficients depending on a wave length. The angle θ is linked to a polar coordinates (α, β) by following relation $z = \cos(\theta) = \sin(\alpha)\sin(\beta)$. Then

$$\frac{dE}{d\alpha} = 2\left(\frac{R}{l}\right)^2 B(0)((1-u-v)\sin^2\alpha + \frac{\pi}{4}u\sin^3\alpha +$$

$$+\frac{2}{3}v\sin^4\alpha) \quad (4)$$

Let angle δ is a visual angular distance from the center of the solar disk. Then the angles α and δ are also interlinked by following relations $l\sin\delta = R\cos\alpha$, $l\cos\delta d\delta = -R\sin\alpha d\alpha$

They allow to proceed from an angle α to an angle δ if necessary.

3. Transparence of an atmosphere for various parts of the solar disk

Illumination of some partial volume in an atmosphere assigned by coordinates (r, r_0) is

$$E = \int_{-\delta'}^{\delta'} F(r, r_0(\delta))\frac{dE}{d\delta}d\delta. \quad (5)$$

Here $F(r, r_0(\delta))$ — transparence of an atmosphere up to partial volume along trajectories of distribution of solar rays;
δ' — angular sizes of the Sun;
$\frac{dE}{d\delta}$ — illumination of a site outside of an atmosphere from a segment of the Sun in a solar meridian with visual angular distance δ.

Let's suppose, that the transparence can be submitted by association

$$F(r, r_0(\delta)) = exp\left(-e^{f(r, r_0(\delta))}\right). \quad (6)$$

Transparence of an atmosphere may be decomposed in a Taylor series on degrees of a smallness $\Delta r_0 = r_0(\delta) - r_0(0)$.
The zero term of expansion is $F_0(r, r_0(\delta)) = exp(-\tau_0)$, where $\tau_0 = e^{f(r, r_0(0))}$.
The first term of expansion

$$F_1(r, r_0(\delta)) = -\tau_0 f'(r, r_0(0))exp(-\tau_0)\Delta r_0.$$

The second term of expansion

$$F_2(r, r_0(\delta)) = \frac{1}{2}e^{-\tau_0}(\tau_0 f'(r, r_0(0)))^2 -$$

$$-\tau_0 f''(r, r_0(0)) - \tau_0(f'(r, r_0(0)))^2)\Delta r_0^2.$$

In all terms of expansion only Δr_0 depends from δ. Therefore, it is possible to bear all remaining factors of each term of expansion for a sign of integration.

In the supposition of rectilinear distribution of solar rays

$$\Delta r_0 = -r(2\cos\phi\sin^2\frac{\delta}{2} - \sin\phi\sin\delta). \quad (7)$$

Here ϕ — angle between radius by vector indicating on dispersing volume, and radius by vector indicating on minimum distance of a ray above a ground surface.

$\frac{dE}{d\delta}$ — an even function from an angle δ. Therefore convolution δ with an odd function from an angle δ (For example, $\sin\delta$) in symmetric limits is equal to zero. Hence, the terms containing only even degrees from a sine of an angle will remain. Let's note the formula for an evaluation of illumination intensity of partial volume of an atmosphere, in which we shall be restricted to the second degrees from $\sin\delta$.

$$E = e^{-\tau_0}\int_{-\delta'}^{\delta'} dE +$$

$$+2\tau_0 e^{-\tau_0} f'(r, r_0(0))r\cos\phi \int_{-\delta'}^{\delta'} \sin^2\frac{\delta}{2}\frac{dE}{d\delta}d\delta +$$

$$+\frac{1}{2}e^{-\tau_0}\tau_0(\tau_0(f'(r, r_0(0)))^2 - f''(r, r_0(0)) -$$

$$-(f'(r, r_0(0)))^2)(r\sin\phi)^2 \int_{-\delta'}^{\delta'} \sin^2\delta\frac{dE}{d\delta}d\delta \quad (8)$$

In all integrals it is possible to pass from an integration on an angle δ to an integration on an angle α. It allows easily calculating integrals analytically. We shall not do it, and we shall conduct a numerical estimation of integrals to reveal the basic regularity of a modification of illumination intensity at limited angular sizes of the Sun from optical depth. Let optical depth varies under the exponential law. Then a second derivation from function $f(r, r_0) = 0$. For the Earth's atmosphere, the first derivative from function f is approximately equal to $0,125$. Let's assume radius the Earth, $r = 6370km$. Radius of the Sun $R = 696000km$. Medium distance from the Sun to the Earth $l = 150mln.km$. Let the atmosphere thickness is $100km$. Then $r\cos\phi = 6370km$, $r\sin\phi = 1133km$

Then an angular sizes of the Sun is $\delta' = 15,95'$
For a wavelength 371 nm.

$$E = E_0(1 + 0.0015\tau_0 + 0.038\tau_0(\tau_0 - 1)). \quad (9)$$

Based on the obtained estimation it is possible to make a following deduction. That for optical depths about one the modification of illumination intensity taking into account limited angular size of the Sun does not exceed 20 of percent. Large optical depths, as a rule, correspond to feebly irradiated stratums and that

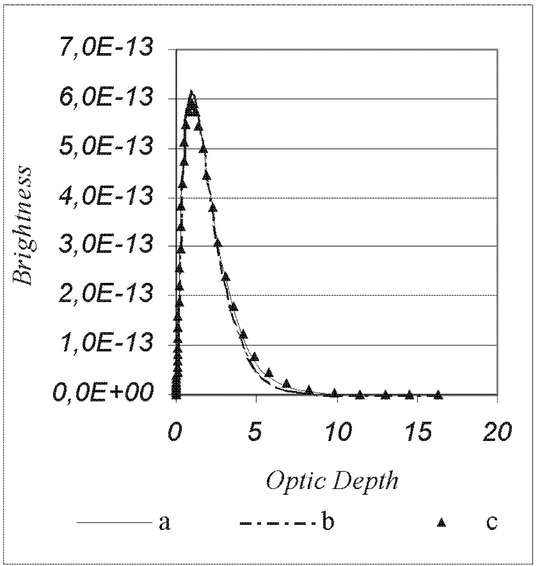

Figure 4: Dependence of various atmospheric slices luminosity on optical length of a trajectory up to a scattering point. a — illumination intensity of scattering volume calculated as a series; b — the illumination intensity calculated neglecting the angular size of the Sun; c — illumination intensity calculated by replacement of the Sun by 7 equidistant radiant points.

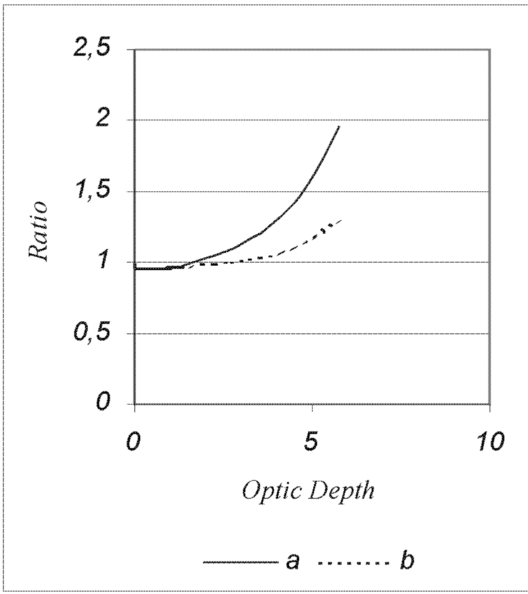

Figure 5: Dependence of the luminosity's ratio on the optical depth for various atmospheric slices. a — the ratio of luminosity's of the scattering volume calculated according to the 7 points model and neglecting angular size of the Sun; b — the ration of luminosity's of scattering volume calculated according to the 7 points model and as the series.

is why their influence on the twilight sky character is small

As an example we shall make calculation of the dependence of twilight atmosphere stratum brightness on the height for a wave length 371 nm and zenith distance in the direction of a sighting 0°, the Sun depth 7°. This wavelength was chosen to position the twilight ray high enough above the surface. It excludes a possible situation, when the Sun is partially closed by a surface of the Earth. Let's conduct calculations for three cases. First case does not take into account angular solar size. Second case takes them into account according to the above mentioned theory and the third one uses Solar model where the Sun is substituted by 7 equally distant points of various intensity placed in the solar meridian (fig. 4). Observing this figure, We are possible to make the next conclusions.

Takings into account angular size of the Sun feebly influence the total brightness of the twilight sky. However, the brightness of some layers of the atmosphere changes significantly. When optic depth is large enough the brightness of the twilight sky significantly increases due to angular size of the Sun (fig. 5).

Methodic given in the paper permits effective calculation of corrections in the field of optical depth about one. These optical depths correspond with the atmospheric layers that contribute main part of the twilight sky brightness. For lager optical depth, the correction may be calculated by taking into account a greater number of terms of the series. However, this way may appear not effective.

The undoubted advantage of this procedure in comparison with the 7-point model is an acceleration of correction computing. Multipoint model increases computing time proportionally (and even more) to the number of points. Our model increases computing time less then twice comparing with the case that neglects angular size of the Sun.

References

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