DENSITY PERTURBATIONS IN A REALISTIC UNIVERSE

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ABSTRACT.

We analyze evolution of density perturbations in a flat or open universe filled in with matter, relativistic particles and possibly cosmological constant. Density perturbations grow very slowly in a universe filled in with low mass neutrinos.

Key words: Cosmology: Big Bang: Early Universe, Density Perturbations

1. Introduction

As we celebrate the 95-th birthday of George Gamow in Odessa, it is quite appropriate, I think, to reflect upon his two important moments of great desperation. It could be a legend that Gamow fled from Odessa to Turkey on a canoe, but even if it is a legend, it is a nice one, worth remembering. One can only imagine the level of desperation which forced him to flee. Several years later, while already in the United States, Gamow desperately tried to convince, first, his friends and associates, and later the whole astronomical community that the universe was created at the Big Bang (Gamow, 1946). After the discovery of the microwave background radiation by Arno Penzias and Robert Wilson in 1964 (Penzias and Wilson, 1965) the Big Bang scenario of the very early evolution of the universe has been universally accepted.

George Gamow was the early driving force of understanding physics of the evolving universe. His ambitious attempt to create all elements that exist in nature at Big Bang failed (Alpher, Bethe, Gamow 1948; Gamow 1948), but the theory of primordial nucleosynthesis later on turned out to provide very important information about the early universe and the constituents of matter. Comparing the observed abundance of light elements, in particular He^4, deuterium, and Li^7 with predictions of the theory of primordial nucleosynthesis we now determine the density of baryonic matter in the Universe (Schramm, Turner 1998). It was noticed by V. Shvartzman (Shvartzman 1969) that the final abundance of He^4 depends on the number of families of relativistic particles present at the epoch on nucleosynthesis. Using this idea it was possible to show that there are only 3 different kinds of weakly interacting neutrinos. Later this result was confirmed by laboratory experiments.

2. Main constituents of a realistic Universe

I think that Gamow would have joined us in the recent period of desperation in cosmology as we straggle to find out what are the main constituents of the Universe.

During the golden period of cosmology in the sixties and seventies it was generally assumed that the universe is filled in with radiation (I really mean with photons and neutrinos) and baryons with electrons (leptons) present to make the total electric charge of the universe equal to zero. As is well known since the time of Friedman, the expansion rate of the universe is determined by the equation

$$H^2(t) = (\frac{\dot{R}}{R})^2 = \frac{8\pi G}{3}\varrho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}, \qquad (1)$$

where R is the scale factor, ϱ is the average density, $k = +1, 0, -1$ is the curvature parameter, and Λ is the cosmological constant. The average matter density can be explicitly written down as $\varrho = \varrho_r + \varrho_m$, where ϱ_r is the average density of all relativistic particles and ϱ_m is the matter density. If matter does not interact with radiation we have

$$\varrho_r R^4 = \text{const}, \quad \text{and} \quad \varrho_m R^3 = \text{const}. \qquad (2)$$

It is useful to introduce so called critical density by the relation

$$H^2 = \frac{8\pi G}{3}\varrho_{crit},$$

where H is the Hubble constant, and the omega parameter $\Omega = \dfrac{<\varrho>}{\varrho_{crit}}$, where $<\varrho>$ is the average matter density. These two basic parameters determine the global properties of the Friedman type homogeneous and isotropic Universe. Actually one can actually define several omega parameters, namely

$$\Omega_r = \frac{\varrho_r}{\varrho_{crit}}, \quad \Omega_m = \frac{\varrho_m}{\varrho_{crit}}, \quad \Omega_c = -\frac{kc^2}{R^2H^2},$$

$$\text{and} \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H^2}. \tag{3}$$

The Friedman equation implies that

$$\Omega_r + \Omega_m + \Omega_c + \Omega_\Lambda = 1. \tag{4}$$

With the help of the omega parameters the Friedman equation can be rewritten as

$$H^2(z) = H_0^2(\Omega_{0r}(1+z)^4 + \Omega_{0m}(1+z)^3 + \Omega_{0c}(1+z)^2 + \Omega_{0\Lambda}), \tag{5}$$

where $1+z = \frac{R_0}{R(t)}$ and corresponding Ω_0 parameters denote their present values.

Attempts to determine the average matter density of the Universe led to an important discovery that there is more matter in the Universe than allowed by the theory of primordial nucleosynthesis and that most baryons do not emit light. I do not want to spend more time discussing these issues since tomorrow Volodia Lukash will present a general overview of the basic cosmological parameters. Let me only mention that $\Omega_{stars} = 0.005 \pm 0.002$ is much smaller than $\Omega_B = 0.045 \pm 0.005$. The best data on average matter density of the universe is provided by flat rotation curves of spiral galaxies, study of motions of galaxies in galaxy clusters, x-ray radiation from clusters of galaxies, and lensing on clusters of galaxies. Large scale flows of galaxies provide an independent estimate of the average matter density. From such dynamical type measurements it follows that $\Omega_m = 0.25 \pm 0.06$.

After very precise measurements of temperature of the microwave background radiation by the COBE satellite $T = 2.726 \pm 0.005$ (Bennett *et. al.* 1994), and laboratory determination that there are only three different types of neutrinos (probably all of very small mass) we have $\Omega_r = 9.5 \cdot 10^{-5}$.

Recent measurements of the Hubble constant and other basic cosmological parameters from observations of distant Type Ia supernovae lead to $H_0 = 65 \pm 2$ km/sMpc and for the first time seriously established that we live in a universe with different from zero cosmological constant and $\Omega_\Lambda \approx 0.75$ (Pelmutter *et.al* 1999; Riess *et. al.* 1999). These measurements imply that the Universe is flat with $\Omega_c = 0$.

3. Evolution of density perturbations

The problem of stability of the Friedman universe was solved by E. M. Lifschitz in 1946 (Lifschitz, 1946). The general relativistic equations describing evolution of small perturbations have been extensively studied since then. I do not want to rederive these equations here. Let me concentrate on the equation describing evolution of density perturbations. Assuming that dark matter particles are non relativistic and pressureless the equation describing density perturbations is usually written in the following form

$$\frac{d^2\Delta}{dt^2} + 2H\frac{d\Delta}{dt} - 4\pi G\varrho_m\Delta = 0, \tag{6}$$

where $\Delta = \delta\varrho/\varrho$.

Using this equation Guyot and Zeldovich (1970) and later independently Meszaros (1974) noticed that radiation strongly suppresses growth of density perturbations in particular in open cosmological models. This fact creates problems since now we have observationally established limits on the amplitude of density perturbations at the epoch of recombination. When density perturbations are adiabatic (what we assume) fluctuations of temperature of the microwave background radiation observed by COBE $\frac{\delta T}{T} \sim 10^{-5}$ restrict the value of $\frac{\delta\varrho}{\varrho}$ at recombination since we have $\frac{\delta\varrho}{\varrho} \approx 3\frac{\delta T}{T}$. In order to create galaxies and large scale structure by $z \approx 10$ the density perturbations should grow during that period by a factor of $\approx 10^4$. This restriction is not a problem anymore since dark matter particles decoupled from thermal equilibrium much earlier than baryons and therefore amplitude of dark matter density perturbations at recombination could be much higher. When baryons cease to interact with radiation after recombination they rapidly fall into gravitational potential wells and soon after recombination we have that $(\frac{\delta\varrho}{\varrho})_B \approx (\frac{\delta\varrho}{\varrho})_{DM}$.

It turns out that equation (6) also holds when $\Lambda \neq 0$. In the general case equations (6) can be conveniently transformed into

$$x(\Omega_r + \Omega_m x + \Omega_c x^2 + \Omega_\Lambda x^4)\Delta'' + (\Omega_r + 3/2\Omega_r x + 2\Omega_c + \Omega_\Lambda x^4)\Delta' - \frac{3}{2}\Omega_m\Delta = 0, \tag{7}$$

where $x = \frac{R(t)}{R_0}$ and $'$ denotes differentiation with respect to x.

In a simpler case, when $\Omega_\Lambda = 0$ this equation was studied by Rozgacheva and Sunyaev (1981), and Rozgacheva (1983) among others. I am not aware of analytical solutions of this equation when $\Omega_\Lambda \neq 0$. Therefor let me present numerical solutions and discuss how the cosmological constant influences evolution of density perturbations. We have numerically solved the equation (7) with the initial condition $\Delta(x = 0.001) = 1$ and therefore the final answer gives the growth factor of density perturbations in their post recombination evolution. If there are three different kinds of massless neutrinos, and the Hubble constant is $H_0 = 65km/(sMpc)$ then $\Omega_r = 0.0001$. Commonly accepted value of the averaged total matter density in the universe (including dark matter) gives $\Omega_m = 0.3$. To include the recent observational estimates of the value of the cosmological constant, we will consider two cases $\Omega_\Lambda = 0.7$, $\Omega_c = 0$ and $\Omega_\Lambda = 0$ and $\Omega_c = 0.7$. The results are shown in Fig. 1.

We enlarge the parameter space by taking into account recent estimates of the mass of neutrinos. The largest possible mass of neutrinos allowed by measurements is in the range 0.1 eV to 0.03eV (Kearns, Kajita and Tostsuka, 1999) so neutrinos are still relativistic particles what leads to Ω_r in the range of 0.002 to 0.007. In Fig. 2 we present results of numerical integration of equation (7) in a flat universe with $\Omega_c = 0$ but with different form zero cosmological constant and curves shown in Fig. 3 represent solutions in an open universe without cosmological constant but with $\Omega_c = 0.698$ and $\Omega_c = 0693$ correspondingly.

4. Conclusions

We confirm the previous results that the density perturbations grow slower in a radiation dominated universe. From Fig. 1, and comparing Fig. 2 and Fig. 3, it is apparent that, with the same Ω_r and Ω_m, density perturbations grow faster in a flat universe with the cosmological constant then in an open universe without the cosmological constant. With the present estimates of the mass of neutrinos the growth of density perturbation in the post recombination period is unacceptably slow.

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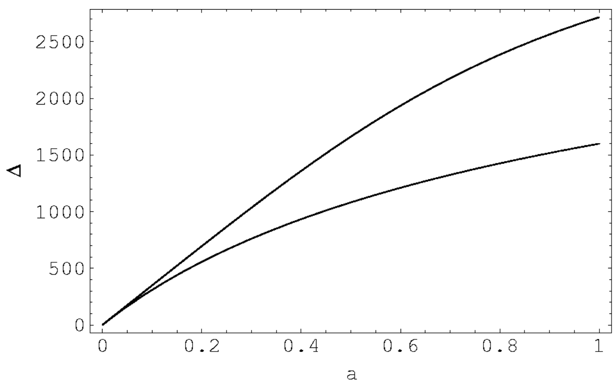

Figure 1: Growth of density perturbations in a Friedman universe with $\Omega_r = 0.0001$, $\Omega_m = 0.3$, and $\Omega_c = 0$, $\Omega_\Lambda = 0.6999$ upper curve, and $\Omega_c = 0.6999$, $\Omega_\Lambda = 0$ lower curve.

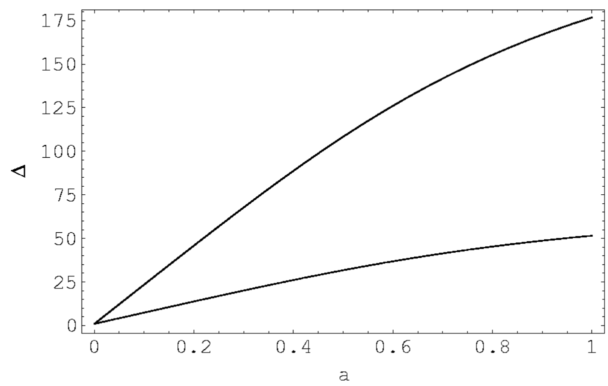

Figure 2: Growth of density perturbations in a Friedman universe with $\Omega_c = 0$, $\Omega_m = 0.3$, and $\Omega_r = 0.002$, $\Omega_\Lambda = 0.698$ upper curve, and $\Omega_r = 0.007$, $\Omega_\Lambda = 0.693$ lower curve.

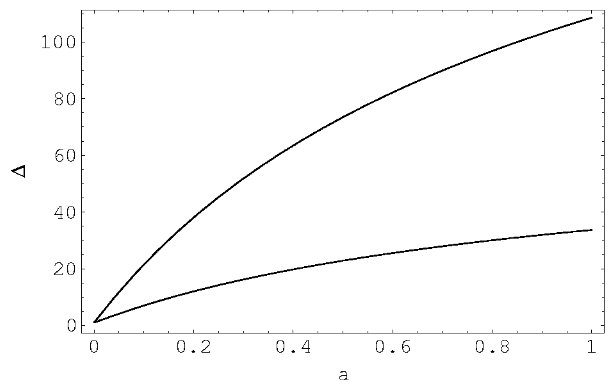

Figure 3: Growth of density perturbations in a Friedman universe with $\Omega_\Lambda = 0$, $\Omega_m = 0.3$, and $\Omega_r = 0.002$, $\Omega_c = 0.698$ upper curve, and $\Omega_r = 0.007$ and $\Omega_c = 0.693$ lower curve.