THERMAL WAVES AND UNSTABLE CONVECTION IN ZZ CETI STARS

N. Falcon, J. Labrador

Departamento de Fisica, FACYT Universidad de Carabobo
Av. Bolivar Norte, Valencia Apto. Postal 129 (casilla12) CP 2001 Venezuela,
nelsonfalcon@hispavista.com

ABSTRACT. We analyze the stability criteria by convection in ZZ ceti stars when the thermal relaxation times are not negligible. Using the Cattaneo law and the Mixing Length Theory in the degenerate regimen, is shown that the heat flux can be propagated by thermal waves and predicts quasi periodic pulses in luminosity. This formalism is consistent with the semi-analytically approaches to temperature variations and several ZZ ceti light curves.

Key words: Stars: interiors, ZZ ceti: convection, stability.

1. Introduction

The thermal properties are responsible of radiation and further evolution of white dwarfs (WD) and ZZ stars. Also, the oscillations and pulsation's (non radial g modes) in DA and ZZ ceti stars are usually attributed to temperature intrinsic variations (Vth et al, 2001; Brickhill, 1992), in which the convection would be an important aspect (Althaus and Benvenuto, 1996; Bergeron et al 1992). Several investigators thinks about these oscillations as "temperatures" waves or heat waves (thought spherical harmonics model in the luminosity variation). However, is necessary to propose some evolutionary pattern of the oscillations observed in the light curves, starting from a formalism based on the usual equations of evolution and stellar structure. But, in standard calculations of the ZZ ceti asterosismology the possibility of the heat propagation by waves is obviated in the energy transport equation. This simplification will be spurious in the degenerate material because, the relaxation time (the time required for to establish the heat flux when one temperature gradient is switched on) is not negligible. In this article, it is described how the energy transport equation and the luminosity in WD stars change if heat waves are taken into account. To make this, the Maxwell-Fourier law is replaced by the Cattaneo causal law (section 2). A general version of variation of the luminosity and the relaxation time based on the Cattaneo equation and the heat waves is presented in section 3. Finally, a short discussion of the results, and its application in ZZ Ceti light curve is given in the last section.

2. The Cattaneo Law and the Energy Transport Equation

If the energy in stars is transported though the stellar layer by radiation, conduction and/or convection, the temperature gradient is given, in a good approximation, in the stellar interior in terms of local values of opacity κ density ρ and energy flux F by the energy transport equation (Kippenhahn and Weigert:1990, p.28; Hansen and Kawaler:1994, p.181.):

$$\frac{dT}{dr} = -\frac{3\kappa\rho F}{4acT^3} \tag{1}$$

where $a = 7,5710^{-15} erg.cm^{-3}K^{-4}$ is the radiation density constant and c is the light velocity. This relation is just Fourier-Maxwell law for energy flux due to thermal conductivity and/or radiative diffusion:

$$\vec{F}(\vec{x},t) = -k\vec{\nabla}T(\vec{x},t) \tag{2}$$

where the coefficient of conduction for the diffusion energy is:$k = \frac{4acT^3}{\kappa}$.

It is well known that Fourier-Maxwell law leads to a parabolic equation for T, according to which perturbations propagate with infinite speed (see Joseph and Preziosi 1989 and references therein). The origin of this non-causal behavior found in Eq. 2 which it is assumed that the energy flux appears at the same time the temperature gradient is switched on. Neglecting the relaxation time (τ) is, in general, sensible thing to do because for most materials it is very small (of the order of 10^{-11} s for the phonon-electron interaction and of the order 10^{-13} s for the phonon-phonon and free electron interaction, at room temperature). There are, however, situations where the relaxation time may not be negligible. For example in neutron star interior the relaxation time τ is the order 10^2 s by temperature the 10^6 K (Herrera and Falcon; 1995a).

The problem of heat propagation for times shorter than the relaxation time has been the subject of lengthy discussions since early work of Maxwell (Joseph and Preziosi 1989 and references therein) and most recently has been introduced in astrophysical scenarios. A heat flux equation leading to a hyperbolic equation is the Cattaneo law (see Joseph and Preziosi,1984; Herrera and Falcn,1995b):

$$\tau \frac{\partial \vec{F}}{\partial t} + \vec{F} = -k\nabla T \tag{3}$$

We assume the Cattaneo law for the temperature gradient similarly to the Eq.(1) we obtain:

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}(\tau\frac{\partial \vec{F}}{\partial t} + \vec{F}) \tag{4}$$

If we replace F by the usual luminosity (L) Eq.(4) can be written as

$$\frac{dT}{dr} = -\frac{3}{16ac}\frac{\kappa\rho}{T^3 r^2}(\tau\frac{\partial L}{\partial t} + L) \tag{5}$$

In case of $\tau \approx 0$ to take notice of the relation (3) and (5) are the "classical" energy transport equation by stellar interior.

3. The Luminosity and the Relaxation Time

The influence and importance of the convection and mixing length theory (MLT) in a study and calculations of the atmosphere model for white dwarf is highly report (Atweh and Eryurt-Ezr, 1992, and references therein). However, the possibility that the time of relaxation is not negligible, in the nucleus of WD, it would bear to the existence of thermal waves. In that case the luminosity change admit quasi periodic variation when the times are less than relaxation time. We now used the Eq. (5) in the convection theory before relaxation (MLT before relaxation) which the luminosity is (Herrera, L. and Falcon, N.; 1995b):

$$L = L^{(d)} f(\chi, \omega) \tag{6}$$

with

$$f(\chi,\omega) = \frac{1}{\omega^2+1} exp\left[\frac{\chi}{2}\left(\omega^2 - 1\right)\right] \tag{7}$$

$$\left[(5+\omega^2)\cos(\omega\chi) - \frac{(3-\omega^2)}{\omega}\sin(\omega\chi)\right]$$

and

$$\omega^2 \equiv \frac{4\tau}{\tau_d} - 1 \tag{8}$$

$$\chi \equiv \frac{t}{2\tau}$$

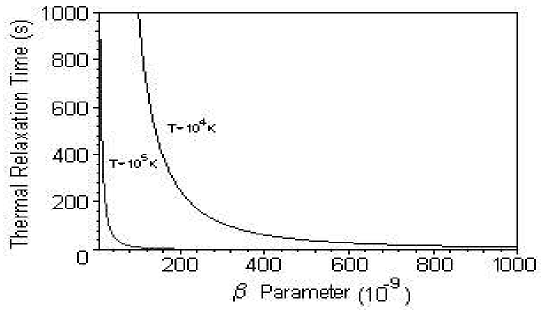

Figure 1: Relaxation Time and Heat Waves Speed to effective temperature interval

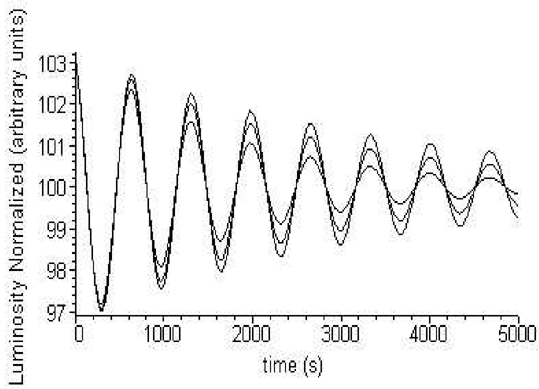

Figure 2: True Luminosity change, for $\omega = 0.94, 0.96, 0.97$ values

This equation (6) connects the standard luminosity (L) of MLT and the luminosity before thermal relaxation ($L^{(d)}$) owing to the heat waves, where f is a function of time t, relaxation time τ and thermal adjust time τ_d. The true luminosity change ($L/L^{(d)}$) is show in the figure 1 for several values of the ω . Note that the luminosity function is a dampen oscillation.

On the other hand, the calculation of the relaxation time can be starting from the relationship (see Herrera and Falcon, 1995a for details):

$$\tau = \frac{k}{V^2 C_V} \approx \frac{10^{-3}T^{-2}}{\beta^2} \tag{9}$$

in the last term it has been used the thermal conductiv-

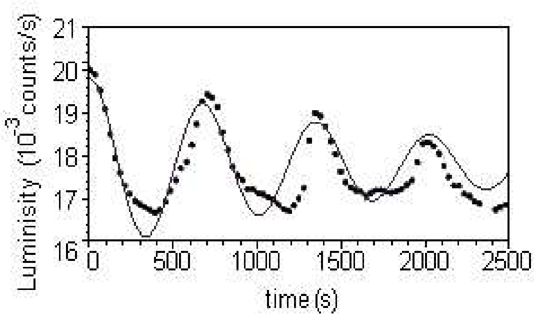

Figure 3: Simulation of the Light Curve of G 29-38

ity (k) for degenerate material (Flower and Itoh, 1979) and the specific heat (C_V) by the Chandrasekhars relationship (Hansen and Kawaler, 1994).

Obviously the relaxation time will depend of the value assumed for the thermal waves speed ($V = \beta c$). The figure 2 shows the relaxation time for diverse values of temperature and several thermal waves speed.

4. Results and Discussion

The luminosity fluctuation, due to the causal propagation of the thermal flow (Eq. 5 and 6) suggest an approximate model for the study of the ZZ Ceti stars. Inside the WD the matter is degenerate and the thermal conductivity is dominated by electrons, therefore the use of the Cattaneo Law is justified.

It can be thought of the existence of statistical fluctuations in the density or in the temperature, in some fluid portion in the stellar deep interior. The aleatory movement of certain convective globule, inside the gradient of temperature, would carry a fluctuation in the temperature and, in consequence a variation in the brightness. The luminosity variation of the convective flow would have, by virtue of the Eq. 6, a dampen oscillatory behavior.

In scales of time comparable to the time of relaxation the stellar total luminosity is due to the contributions of the radiative flow (approximately constant) more the contribution of the convective flow (which is damped oscillatory). According to this model the periods of the luminosity fluctuations would be of the order of the thermal relaxation time. Also, because the convective flow is only an fraction of the total thermal flow, then the luminosity variations would be dampen and small width regarding the intrinsic luminosity. The behavior as erratic function of the ZZ ceti light curves would be explained in term of the sum of several convective fluctuations along time.

Several convective flows sequentially, each one due to some specific thermal fluctuation, they could reproduce the light curves of some ZZ ceti stars. Indeed, it is shown in the figure 3, the modeling of the light curve of the rapid variable G29-38 (data experimental due to McGraw, 1977). Here we used that the relaxation time in order to 1000 second and the thermal wave speed in order to sound speed in degenerate material.The thermal adjust time is account in order to 12 second.

The group of Eq. 6-9 can be used for the modeling of other light curves, using conveniently it values of relaxation time and of the thermal adjustment time. For that should be considered the speed of the thermal waves and the effective temperature by means of the figure 2.

The presented ideas could be good in order to connect the usual stellar evolution formalism with the quasi-analytical approaches (distribution of temperature surface like sum of spherical harmonics) in the study about ZZ ceti light curve.

References

Atweh Z.M. & Eryurt-ezr D.: 1992, *Ap.S.S.,* **189**, 27.

Althaus L.G. & Benvenuto: 1996, *MNRAS*, **278**, 981-984.

Brickhill A.J. : 1992, *MNRAS*, **259**, 519-528.

Bergeron P., Wesemael F. & Fontaine G.: 1992, *Ap. J.*, **387**, 288.

Flowers E. & Itoh N.: 1979, *Ap.J.*, **230**, 847-858.

Hansen C.J. & Kawaler S.D.: 1994, *"Stellar Interiors Physical Principles, Structure and Evolution"*, Springer-Verlag, N.Y.

Herrera L. & Falcon N.: 1995, *Ap.S.S.*, **229**, 105.

Herrera L. & Falcon N.: 1995, *Ap.S.S.*, **234**, 139.

Joseph D. & Preziosi L.: 1989, *Rev. Mod. Phys*, **61**, 41.

McGraw J.T.: 1977, *PhD Thesis. University of Texas at Austin.*

Kippenhahn R. & Weigert A.: 1990, *"Stellar Structure and Evolution"*, Springer-Verlag, Berlin.

Vath H., Koester K., Kepler S.O. & Robinson E.L.: 2001, *http : //www.astrophysik.uni − kiel.de/ pershome/supas097/reprints/proc$_w$d96/proc$_w$d96. html..*