INFLATION THEORIES AND PLANK MASS PROBLEM

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ABSTRACT. A method of producing large value of Plank mass with aids of scalar field theory is being investigated. Correspondingly, inflation models are being discussed from this point.

Special combinations of model parameters that ensure duration of inflation being greater than 60 e-foldings were derived. By means of numerical modeling we verified analytical approximations and give illustrative picture of evolution of fields and Hubble constant H for those periods of inflation, when it is not possible to perform analytical calculations.

Keywords: Cosmology, Inflation, Plank Mass, Numerical Calculations.

Introduction

Recently the problem of large value of Planck mass attracts special attention among researchers. Different models are being proposed in order to achieve effective value of Planck mass being equal to $M_{Pl} = 10^{19} GeV$ by means of special mechanisms, while real value of ${M_{Pl}}^*$ is considered rather small (e.g., in: Arkani-Hamed et al. 1998, authors attempted to solve the indicated problem with aids of compact extra dimensions).

In the present investigation authors attempted to produce correct effective value of Plank mass from smaller ${M_{Pl}}^*$ by introducing scalar field φ into the action S. One cannot argue that inflation models with appropriate corrections to Einstein equations taken into account are to be thoroughly discussed in the case. Here we tested one simple inflation model with one real scalar field with inflation potential $V = \lambda(\varphi^2 - v^2)^2$. We need to point out one's attention to the fact that scalar-tensor theories of gravitation are of great interest among researches (e.g., Faraoni, 2004), for example, well-known Brans-Dicke theory. Furthermore, there were attempts to use theories of this type even in theories of inflation (Yoshimura, 1991).

1. Model Description

As it was mentioned above, our approach is to introduce scalar field φ in the action. Thus the action S is as follows:

$$S = \int \sqrt{-g} d^4 x e^{\frac{\phi}{M}} \left(\frac{1}{16\pi\hat{G}} R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right),$$

where

$$dS^2 = -dt^2 + a^2(t) \left(dx^2 + dy^2 + dz^2 \right),$$

$$V(\phi) = \lambda(\phi^2 - v^2)^2$$

(λ, v and M are model parameters). One can see, that in the case $M_{Pl}^2 = \frac{1}{16\pi\hat{G}} e^{\frac{v}{M}}$ (at the point $\phi = 0$), or $M_{Pl}^2 = {M_{Pl}^*}^2 \, e^{\frac{v}{M}}$. Actions of this type (with so-called "dilaton field") were studied earlier as a solution for different problems in cosmology (see e.g., Ellis et al., 2000). The main target of the research is to find out the highest possible magnitude of ratio $\frac{v}{M}$ (and consequently the lowest value of M_{Pl}^*) that still allows inflation processes to occur in the model, and to derive the spectrum of model parameters that makes inflation last more than 60 e-foldings.

Einstein equations (and equations of motion) are:

$$\frac{3}{16\pi\hat{G}} \left(\frac{\dot{a}}{a} \right)^2 - \frac{1}{2} \left(\dot{\phi}^2 + V(\phi) \right) + \frac{3}{16\pi\hat{G}} \left(\frac{\dot{a}}{a} \right) \left(\frac{\dot{\phi}}{M} \right) = 0;$$

$$\frac{1}{16\pi\hat{G}} (\dot{a}^2 + 2\ddot{a}a) + \frac{1}{2} a^2 \left(\dot{\phi}^2 - V(\phi) \right) + \frac{1}{16\pi\hat{G}} \times \\ \times \left(a^2 \frac{\ddot{\phi}}{M} + a^2 \frac{\dot{\phi}^2}{M^2} + 2\dot{a}a \frac{\dot{\phi}}{M} \right) = 0;$$

$$\frac{6}{16\pi\hat{G}} \frac{1}{M} (\ddot{a}a + \dot{a}^2) - 6\dot{a}a\dot{\phi} - 2a^2\ddot{\phi} - \\ -a^2 \frac{1}{M} \left(\dot{\phi}^2 + V(\phi) \right) - a^2 \frac{\delta V}{\delta \phi} = 0.$$

After some simple algebra it is convenient to transform the equations to more "familiar" form (with all terms with M grouped together):

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{1}{\sqrt{2}M} \left[V(\varphi) + \frac{\dot{\varphi}^2}{2} - \frac{3}{8\pi\hat{G}} \left(2H^2 + \dot{H} \right) \right] = 0;$$

$$3H^2 = 8\pi\hat{G} \left(\frac{\dot{\varphi}^2}{2} + V(\varphi) \right) - \frac{1}{\sqrt{2}M} 3H\dot{\varphi};$$

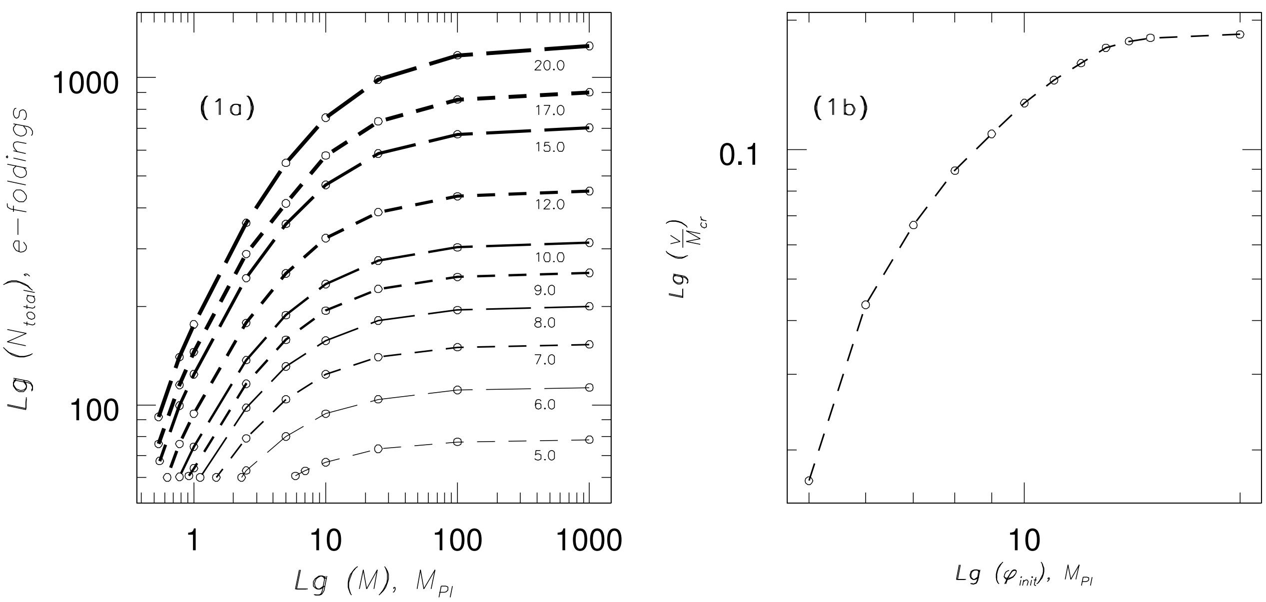

Figure 1: Fig. (1a) shows "$\varphi = \varphi_{init}$-slice" of $N_{total}(M_{cr}, \varphi_{init}), e - foldings$. Numbers to the right-side of each curve gives appropriate value of φ_{init}, M_{Pl}. Fig. (1b) shows $\frac{v}{M_{cr}}(\varphi_{init})$, which is "$N_{total} \approx 60$" slice.

$$-2\dot{H} = 8\pi\hat{G}\dot{\varphi}^2 + \frac{1}{\sqrt{2}M}\left[\ddot{\varphi} - H\dot{\varphi} + \frac{\dot{\varphi}^2}{\sqrt{2}M}\right]$$

(here $\phi = \frac{\varphi}{\sqrt{2}}$, Hubble constant H is $H = \frac{\dot{a}}{a}$). We estimate duration of inflation in "e-foldings":

$$N_{total} = \int_{t_{init}}^{t_{final}} H \quad dt.$$

2. Numerical Simulation

It is clear that one cannot solve these equations analytically (at least, not with relatively high $\frac{v}{M}$. Let's rewrite our equation set in the following manner:

$$\dot{x}_0 = x_1;$$

$$\dot{x}_1 = -a_1 V_{x_0}(x_0) - a_2 V(x_0) - a_3\tilde{x}_4^2 - 3a_4\tilde{x}_4 x_1 - a_5 x_1^2;$$

$$\dot{x}_4 = b_1 V(x_0) + b_2 x_4^2 + b_3 x_4 x_1;$$

$$\dot{x}_5 = \tilde{x}_4 x_5,$$

where

$$a_1 = \frac{32M^2\pi\hat{G}}{k}; \quad a_2 = \frac{16\sqrt{2}M\pi\hat{G}}{k};$$

$$a_3 = \frac{12\sqrt{2}M}{k}; \quad a_4 = \frac{32M^2\pi\hat{G} - 1}{k};$$

$$a_5 = \frac{8\sqrt{2}M\pi\hat{G} + 24\sqrt{2}M\pi\hat{G} + \frac{3}{\sqrt{2}M}}{k};$$

$$b_1 = 8\pi\hat{G}\frac{\sqrt{2}M + k + 1}{k}; \quad b_2 = 3\frac{2-k}{k};$$

$$b_3 = \frac{32M\pi\hat{G} - \frac{3}{\sqrt{2}M}k}{k};$$

$$k = 32M^2\pi\hat{G} + 3;$$

$$\tilde{x}_4 \equiv H = \sqrt{\frac{8\pi\hat{G}}{3}\left[\frac{x_1^2}{2} + V(x_0)\right] + \frac{x_1^2}{4M^2}} - \frac{x_1}{2M};$$

and $x_0 \equiv \varphi$; $\quad x_1 \equiv \dot{\varphi}$; $\quad x_4 \equiv H$; $\quad x_5 \equiv a$.

The authors performed numerical calculations by taking advantage of classical IV-order Runge-Kutta algorithm with adaptive step-correction for solving linear differential equations.

An opportunity of real-time processing and data visualization was taken into advantage to perform primary analysis of evolutionary tracks without necessity to use side-built graphics packages. This considerably reduced overall time of treatment of evolutionary tracks, since authors were able to make immediate decisions concerning applicability of sets model parameters being currently chosen.

3. Preliminary Results

In the present investigation we derived the spectrum of model parameters, that allows inflation to last 60 e-foldings or more. As a beginning, we tried to study conventional scales of energy (with Plankian scale 10^{19} GeV). It's obvious that in this case M^*_{Pl} is to be of the same order as well. The λ parameter of the model was always being chosen according to the idea that initial value of the potential V_{init} is supposed to be $V_{init} \sim M^4_{Pl}$; we fixed v parameter to $v = 0.1M_{Pl}$.

Table 1: Critical values of M

φ_{init}, M_{Pl}	λ	V, M^4_{Pl}	M_{cr}, M_{pl}
20,00	6,250E-06	1,600E+05	0,54
15,00	1,975E-05	5,062E+04	0,55
14,00	2,603E-05	3,841E+04	0,56
13,00	3,502E-05	2,856E+04	0,58
12,00	4,823E-05	2,073E+04	0,63
11,00	6,831E-05	1,464E+04	0,69
10,00	1,000E-04	9,998E+03	0,78
9,00	1,525E-04	6,559E+03	0,92
8,00	2,442E-04	4,095E+03	1,12
7,00	4,167E-04	2,400E+03	1,50
6,00	7,720E-04	1,295E+03	2,30
5,00	1,601E-03	6,245E+02	5,90

We found that with lowering M the duration of inflation falls down as well, which makes it necessary to raise initial value of field φ_{init}. The lowest possible value of M parameter for each φ_{init} with $N_{total} \approx 60$ e-foldings is called M_{cr}; appropriate values of parameters are listed in Table 1. We built two slices of $N_{total}\,(M_{cr}, \varphi_{init})$ dependance: "$\varphi = \varphi_{init}$" and "$N_{total} \approx 60$" (see Fig. (1a)-(1b)). This allows one to easily comprehend the results given in Table 1.

One can see, that the lowest available value of M_{cr} is $M_{cr} \approx 0.54$, thus $M^2_{Pl}/M^2_* \approx e^{0.1852} \sim 1$. In general, it means, that in the boundaries of this model one cannot lessen the value of M_{Pl} by means of introducing scalar field in the action - at least, at these scales of energy. Nevertheless, it is of great interest to explore behavior of fields, Hubble constant, etc. with somewhat more realistic inflation potentials. Another subject for research is to examine lower scales of energy, thus making M^*_{Pl} of lower magnitude as well.

As an outlook for the further investigations, we are planning to perform this study for the hybrid inflation model, which includes two interacting scalar fields, thus allowing additional opportunities for fine-tuning.

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