EVALUATION OF OUTER CORE VISCOSITY INFLUENCE ON THE EARTH FORCED NUTATION

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ABSTRACT. On the base of the combined method, consisting of the Sasao, Okubo, Saito (1980) approach with the iteration procedure, allowing couple finite element method with the solution of the Laplace equation, it have evaluated the influence of outer core viscosity on the components of the earth forced nutation. It was derived, that outer core viscosity actually doesn't have influence on the in phase nutation components, but this influence can reach some percents for out of phase nutation components, if we suggest the average value of the outer liquid core viscosity is the order of 10^{10} Pas.

Key words: Nutation: viscosity of outer core: finite element method.

1. Introduction

Still, it is absent certain opinion about the value of outer core viscosity and it's distribution. The evaluations of the average outer core dynamic viscosity, detected by different methods: astro-geodesic, seismic, geomagnetic, methods of melting metals, have scattering from 10^{-1} to 10^{14} Pas. Probably, this scattering have place due to different approaches. But analysis of the approaches permits pick out the order of the most coordinated average viscosity value about 10^{10} Pas. As for law of outer core viscosity distribution, in present time dominates conception: outer core consists on melting metal of the high-viscosity from 10^4 near mantle boundary to 10^{14} Pas into inner core boundary layer of 100 km width [1]. In the Sasao, Okubo, Saito (1980) [2] generalization the tidal variations of the moments of inertia of the elastic mantle and of the fluid core are accounted being parameterized with the help of so called compliances $\kappa_c, \gamma_c, \beta_c$. This compliances may be expressed in terms of the static and dynamic Love numbers k of the second order for the earth and the outer core respectively. Such parametrization also permits to calculate the dissipative effects of the outer core by using imaginary parts of the compliances [3]. In this paper on the base of the combined method [4-6] we have detected the forced nutation of the rotating, self gravitating earth, which consists on elastic mantle, viscosity outer core and solid inner, without calculation of the ocean and atmosphere loadings.

2. Formulation of the problem

In the common case, the molecular liquid viscosity be quit of the shear viscosity η_s and volume viscosity η_v. Because the shear viscosity η_s plays the main role in the dissipative processes in the case of the toroidal oscillations of the liquid core due to precession-nutation movements of the earth, so we shall consider only this viscosity component. As the influences of the inner core dynamics on the Love number k of the second order are vanishing small, so using of the SOS (1980) approach for detecting of the nutation parameters is enough. Following this approach, we shall define the complex compliances of the earth, consisting on elastic mantle, viscous liquid outer core and inner solid core in the spherical quasi-statical approximation. In this case we have two quasi-statical equations for elastic mantle and viscous liquid core, presented in the Tisserand reference system (X, Y, Z):

$$0 = grad(V_c + \phi_n + V_1 u_R g(R)) -$$
$$-div\vec{u}gradW + \frac{1}{\rho}divP_1;$$

$$0 = grad(V_e + \phi_n + \phi_c + V_1) + \frac{1}{\rho}divP_2. \qquad (1)$$

Where $\vec{u}$ - displacement vector; $V_e = \gamma\Omega^2(xzcos\sigma t + yzsin\sigma t)$ - tesseral part of tidal wave potential, γ - it's amplitude; $\phi_n = -\epsilon\Omega^2(xzcos\sigma t + yzsin\sigma t)$ - changing of centrifugal potential due to nutation; ϵ - polar motion radius of tidal wave; $V_1 = \gamma_1\Omega^2(xzcos\sigma t+yzsin\sigma t)$ - changing of gravitation potential due to earth deforming, γ_1 - it's amplitude; $\phi_c = -\beta\Omega^2(xzcos\sigma t+yzsin\sigma t)$ - changing of centrifugal potential due to earth core rotation; β - polar motion radius of core rotation; Ω - angle earth velocity; W - self gravitation potential; ρ - density; σ - frequency of the tidal wave; R - earth point radius; u_R - radial displacement; $g(R)$ - gravity acceleration; P_1, P_2 - changing of stress tensors in the elastic mantle and viscous liquid core respectively. Let us suggest, that oscillations into the liquid core are going with the tidal wave frequency σ, also suggest the absence of the earth surface loads, make up Lagrange functionals. Those functionals express the full energy of the elastic mantle and liquid core in the cylindric coordinate system (z, r, φ), here axis r coincide with the Tisserand axis Z:

$$E_1 = \pi \int_{F_s} [C_1(\varepsilon_{zz}^2 + \varepsilon_{rr}^2 + \varepsilon_{\varphi\varphi}^2) + 4C_2\varepsilon_{zr}^2 + \\ + 2C_3(\varepsilon_{zz}\varepsilon_{rr} + \varepsilon_{zz}\varepsilon_{\varphi\varphi} + \varepsilon_{rr}\varepsilon_{\varphi\varphi})]rdzdr - \\ - \pi \int_{F_s} [2(\gamma - \epsilon + \gamma_1)\Omega^2 rw + w(\frac{\partial w}{\partial z}cos\alpha + \\ + 2\frac{\partial u}{\partial z}sin\alpha)g(R) + 2w(wcos\alpha + \\ + 2usin\alpha)g'_R cos\alpha - 2w(\frac{\partial w}{\partial z} + \\ + 2(\frac{\partial u}{\partial r}))g(R)cos\alpha + 2(\gamma - \epsilon + \gamma_1)\Omega^2 zu + \\ + u(2\frac{\partial w}{\partial r}cos\alpha + \frac{\partial u}{\partial r}sin\alpha)g(R) + 2u(2wcos\alpha + \\ + usin\alpha)g'_R sin\alpha - 2u(2\frac{\partial w}{\partial z} + \\ + \frac{\partial u}{\partial r} + \frac{u}{r})g(R)sin\alpha]\rho rdzdr; \quad (2)$$

$$E_2 = \pi \int_{F_s} [C_4(\varepsilon_{zz}^2 + \varepsilon_{rr}^2 + \varepsilon_{\varphi\varphi}^2) + 4C_5\varepsilon_{zr}^2 + \\ + 2C_6(\varepsilon_{zz}\varepsilon_{rr} + \varepsilon_{zz}\varepsilon_{\varphi\varphi} + \varepsilon_{rr}\varepsilon_{\varphi\varphi})]rdzdr - \\ - \pi \int_{F_s} [2(\gamma - \epsilon - \beta + \gamma_1)\Omega^2 rw + \\ + 2(\gamma - \epsilon - \beta + \gamma_1)\Omega^2 zu]\rho rdzdr. \quad (3)$$

Here $C_1 = K + \frac{4\mu}{3}, C_2 = \mu, C_3 = K - \frac{2\mu}{3}$ - real coefficients; $C_4 = K_f + \frac{4i\sigma\eta_s}{3}, C_5 = i\sigma\eta_s, C_6 = K_f - \frac{2i\sigma\eta_s}{3}$ - complex coefficients; μ, K - shear and compress modules of the elastic mantle; K_f - compressibility of the liquid core; η_s - shear dynamic viscosity of the liquid core; i - imaginary unit; ε_{ij} - deformation components; w, u - displacement components along z, r respectively; F_s - meridional cross section area of the earth; $cos\alpha = \frac{z}{R}, sin\alpha = \frac{r}{R}$. For resolving the system equations (1), without calculation of the surface loads, the finite element method is using. This method is based on the variational Lagrange principal, expressing the minimum of the full system energy, tends to resolving system of variational equations, such as:

$$\delta E_1 = 0; \delta E_2 = 0. \quad (4)$$

The detailed description of the finite element resolving problem is presented in the publications [4,5]. But the finite element only, doesn't permit to calculate the earth gravity field relaxation due to it's deforming. For resolving this problem we will use Wu (2004) approach [7]. As earth deforming potential V_1 is harmonic function, so it must satisfy to Laplace equation. Another hand, exactly resolving of the Laplace equation can be defined from the radial displacements u_r on the heterogeneity layer boundaries of the earth. The radial displacements can be detected by the finite element method [4]. The combined resolving of the problem is realizing during for iteration process:

1)firstly, the finite element problem is realizing for $V_1 = 0$, as a result, the radial displacements u_r are defined;

2)the meanings of the earth deforming potential V_1 are defining from the detected radial displacements u_r on the base of the Wu formalism [7];

3)then, presented here procedure repeats with the calculation of the defined meanings of the potential V_1;

4)the iteration process is continuing till of the convergence of the results of the resolving problem; as calculation shows, the convergence have been gained after 4 - 6 iterations.

3. The definition of the earth forced nutation with the calculation of the outer core viscosity

The retrograde and prograde circular nutation components in phase and out-of-phase are detected by formulas [8]:

$$\eta^+ = -\frac{1}{2}(\epsilon_r + \Psi_r sin\epsilon_0)(\frac{\eta^+}{\eta_r^+}) = B_R^+ - iB_I^+; \quad (5)$$

$$\eta^- = -\frac{1}{2}(\epsilon_r - \Psi_r sin\epsilon_0)(\frac{\eta^-}{\eta_r^-}) = B_R^- - iB_I^-. \quad (6)$$

Where ϵ_0 - ecliptic obliquity; B_R^+, B_R^- and B_I^+, B_I^- - are retrograde and prograde circular nutation components in phase and out-of-phase respectively; the meanings of nutation for rigid earth in the obliquity ϵ_r and in the longitude Ψ_r were took from Kinoshita (1977) theory [9]. The relative amplitudes of the retrograde $\frac{\eta^+}{\eta_r^+}$ and prograde $\frac{\eta^-}{\eta_r^-}$ circular nutation components were defined on the base of the combined method [4-6] with the calculation of the crossing from polar motion to nutation coordinates:

$$\eta(\sigma) = \frac{A_f}{A_m}\frac{e\Omega^2}{\sigma(\sigma - \sigma_0)}[1 - \frac{\gamma_c}{e} + \frac{\gamma_c - \kappa_c}{e}\frac{\sigma + \Omega}{\Omega}]\gamma + \\ + [\frac{e\Omega^2}{\sigma(\sigma + \Omega)} - \frac{\kappa_c\Omega}{\sigma}]\gamma; \quad (7)$$

$$\sigma_0 = -\frac{A}{A_m}(e_f - \beta_c)\Omega - \Omega; \quad (8)$$

$$\kappa_c = \frac{k_o R_e^5 \Omega^2}{3GA}; \gamma_c = \frac{k_0^f R_f^5 \Omega^2}{3GA_f}; \beta_c = \frac{k_1^f R_f^5 \Omega^2}{3GA_f}. \quad (9)$$

Here e, e_f - ellipticities of the earth and outer core respectively; A, A_m, A_f - the main moments of inertia of the earth, mantle and outer liquid core; σ_0 - the near diurnal resonance of the liquid core frequency; R_e, R_f - radiuses of the earth and liquid core respectively; G - gravitation constant. The complex, due to liquid core viscosity, static Love numbers of the second order for the earth and liquid core: k_0, k_o^f were defined on the base of above presented iteration procedure under static conditions: $\gamma - \epsilon = 1, \beta = 0$, from the formulas: $k_0 = \gamma_1(R_e), k_0^f = \gamma_1(R_f)$. There $\gamma_1(R_e), \gamma_1(R_f)$ - are complex amplitudes of the gravitating potential V_1 on the earth and liquid core surfaces respectively. The complex dynamic Love numbers: k_1, k_1^f were defined analogically under dynamic conditions: $\gamma - \epsilon = 0, \beta = 1$. Then complex compliances: $\kappa_c, \gamma_c, \beta_c$ were detected by the formulas (9). As for typical liquid core viscosity values the imaginary part of the β_c parameter become vanishing small, so we have neglected by one and have considered only real part of the liquid core resonance frequency σ_0 (8). At the detection iteration process ten layer earth model, obtaining on the base of the standard earth "PREM" - model, was used. The obliquity and longitude nutation components in phase and out-of-phase can be defined by the formulas:

$$\begin{aligned} \epsilon_R = -B_R^- - B_R^+; \Psi_R sin\epsilon_0 = B_R^- - B_R^+; \\ \epsilon_I = B_I^- - B_I^+; \Psi_I sin\epsilon_0 = B_I^- + B_I^+. \end{aligned} \quad (10)$$

The meanings of the retrograde and prograde circle nutation components in phase and out-of-phase for: 18.6 - year, annual, semiannual and fortnightly terms in milliseconds are presented in the Table 1. The meanings were obtained by combined method for rotating, self gravitating earth, consisting on the elastic mantle, viscosity outer liquid core and rigid inner core, without calculation of the ocean and atmosphere loadings. It was considered 2 variants:

a)the variant of the homogeneous viscosity liquid core with the characteristic shear dynamic viscosity equaled 10^{10} Pas;

b)the variant, which calculates the outer-inner core boundary layer of 100 km width, with the viscosity value of the order 10^{14} Pas, in the rest part of the liquid core we choose the average viscosity value of the 10^8 Pas.

For comparison, there are respective meanings of the nutation components, presented by modern nutation theory - MHB (2000) [10], at this table. The comparison shows, that viscosity effect of the liquid core actually doesn't have influence on the in phase nutation components, another hand this influence may reach some percents for the out-of-phase nutation components if we suggest the average liquid core viscosity value is equal 10^{10} Pas. Also the comparison of the results, obtaining on the base of two above considering variants (a) and (b), tends to suggestion, that value of the order 10^{10} Pas is the top boundary of the average liquid core viscosity.

Table 1: The retrograde and prograde circular nutations in phase and out-of-phase, presented for main nutation terms in milliseconds.

Parameters	(a)	(b)	MHB-2000
phase	in	in	in
	out of	out of	out of
18.6-year			
η^+	-1181.1903	-1181.1903	-1180.3727
	-0.0028	-0.0005	-0.1035
η^-	-8021.8201	-8021.8201	-8024.8023
	0.0223	0.0037	1.4295
annual			
η^+	-31.3005	-31.3005	-33.0475
	0.0047	0.0008	0.3395
η^-	25.6561	25.6561	25.6475
	0.0005	0.0001	0.1365
semiannual			
η^+	-24.5802	-24.5802	-24.5590
	-0.0014	-0.0002	-0.0453
η^-	-549.0812	-549.0812	-548.5020
	-0.0151	-0.0025	-0.5043
fortnightly			
η^+	-3.6401	-3.6401	-3.6417
	-0.0001	-0.0000	-0.0153
η^-	-94.0103	-94.0103	-94.1722
	0.0047	0.0008	0.1477

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