NEUTRINO IN GRAVITATIONAL FIELD

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ABSTRACT. The spin-gravity coupling for highly relativistic fermions according to the classical Mathisson-Papapetrou and quantum Dirac equations is considered. It is stressed that the behavior of fermions in the highly relativistic regime in the gravitational field is significantly different as compare to usual situations. Possible corrections to the known general relativistic Dirac equation for more adequate description of fermions in strong gravitational fields are discussed. Some numerical estimates for neutrinos with nonzero mass that is interesting in astrophysics are presented.

Key words: Dirac and Mathisson-Papapetrou equations: highly relativistic spin-gravity coupling: Lorentz-violating spinor.

The equation for a quantum particle with spin 1/2 in the gravitational field is known since 1929 when the usual Dirac equation was generalized for curved spacetime in general relativity [1]. The corresponding (in a certain sense) equations for a classical (nonquantum) spinning particle are eight years "younger" and are known as Mathisson-Papapetrou (MP) equations [2].

The traditional form of MP equations is

$$\frac{D}{ds}\left(mu^\lambda + u_\mu \frac{DS^{\lambda\mu}}{ds}\right) = -\frac{1}{2}u^\pi S^{\rho\sigma} R^\lambda{}_{\pi\rho\sigma}, \quad (1)$$

$$\frac{DS^{\mu\nu}}{ds} + u^\mu u_\sigma \frac{DS^{\nu\sigma}}{ds} - u^\nu u_\sigma \frac{DS^{\mu\sigma}}{ds} = 0, \quad (2)$$

where $u^\lambda \equiv dx^\lambda/ds$ is the particle's 4-velocity, $S^{\mu\nu}$ is the tensor of spin, m and D/ds are, respectively, the mass and the covariant derivative with respect to the particle's proper time s, and $R^\lambda{}_{\pi\rho\sigma}$ is the Riemann curvature tensor. The MP equations are considered with some supplementary condition and most often the condition $S^{\mu\nu}u_\nu = 0$ or $S^{\mu\nu}P_\nu = 0$ are used, where P_ν is the particle's 4-momentum (in general, P_ν is not parallel to u_ν). In the linear spin approximation the corresponding solutions of the MP equations coincide at the two these conditions. It is important that the MP equations can be used for the investigation of spinning particle motions with any velocity relative to the source of the gravitational field (for example, Schwarzschild's or Kerr's black hole), up to the speed of light, similarly as the geodesic equations are used for a fast moving spinless particle.

The MP equations can be written in terms of the tetrad quantities comoving with the spinning particle [3]. It follows from this representation that in the concrete case of particle motion in Schwarzschild's background, when the particle's spin is orthogonal to the plane determined by the direction of particle motion and the radial direction, the absolute value of the particle 3-acceleration relative to geodesic free fall as measured by the comoving observer is proportional to γ^2, where γ is the relativistic Lorentz factor as calculated by the tangential particle velocity relative to the Schwarzschild mass. It means that there is significant difference in the reaction of a spinning particle on the gravitational field when its velocity is much less than the velocity of light (with γ of order 1) and is very close to this velocity ($\gamma \gg 1$). From the point of view of the comoving observer, the deviation of a spinning particle from the geodesic free fall is caused by the gravitomagnetic components of the moving Schwarzschild source.

Moreover, the strong action of gravity on a highly relativistic spinning particle is not only in the expression for the local accelerate of this particle relative to a spinless particle but in the trajectories of the spinning particle as compare to the corresponding geodesic trajectories as well [4]. Indeed, if the spinning particle posses the velocity relative to Schwarzschild's mass which corresponds to the γ-factor of order $1/\sqrt{\varepsilon_0}$, where $\varepsilon_0 \equiv |S_0|/(mM)$ (here $|S_0|$ and m are, respectively, the values of the particle's spin and mass; M is the Schwarzschild mass), its orbits that begin in the space region near $1.5r_g$ can significantly differ from the corresponding geodesic orbits. This result follows from the MP equations in the linear spin approximation, i. e. is common at the conditions $S^{\mu\nu}u_\nu = 0$ and $S^{\mu\nu}P_\nu = 0$. In addition, according to the exact MP equations at condition $S^{\mu\nu}u_\nu = 0$ the essentially nongeodesic highly relativistic orbits of the spinning particle are allowed for the radial coordinate r which is much greater than $1.5r_g$ and the necessary value of the γ-factor is proportional to $\sqrt{r}$. Some of these orbits show the significant attractive action of the spin-gravity coupling on a particle and others are caused by the significant repulsive action, dependently on the spin orientation.

By the numerical estimates for an electron in the gravitational field of a black hole with three of the Sun's mass the value $|\varepsilon_0|$ is equal to 4×10^{-17}. Then the necessary value of the γ-factor for the realization of some highly relativistic circular orbits by the electron near this black hole is of order 10^8. This γ-factor corresponds to the energy of the electron free motion of order 10^{14} eV. Analogously, for a proton in the field of such a black hole the corresponding energy is of order 10^{18} eV. For the massive black hole those values are greater: for example, if M is equal to 10^6 of the Sun's mass the corresponding value of the energy for an electron is of order 10^{17} eV and for a proton it is 10^{21} eV. Note that for a neutrino near the black hole with three of the Sun's mass the necessary values of its γ-factor for motions on the highly relativistic circular orbits correspond to the neutrino's energy of the free motion of order 10^5 eV. If the black hole's mass is of order 10^6 of the Sun's mass, the corresponding value is of order 10^8 eV. So, some particles in cosmic rays posses a sufficiently high γ-factor for motions on the significantly nongeodesic orbits near black holes. Concerning neutrinos, perhaps, the corresponding effects of the highly relativistic spin-gravity coupling can be registered by the IceCube neutrino detector.

By the way, concerning the possible behavior of a highly relativistic neutrino in Schwarzschild's background we point out paper [5] where some solutions of the MP equations at the condition $S^{\mu\nu}P_\nu = 0$ are studied. The conclusion is formulated that gravitation can accelerate neutrinos to the superluminal motion due to their spin. In this context we stress that 1) this effect is not allowed by the MP equations at the condition $S^{\mu\nu}u_\nu = 0$ and 2) it is shown in [6] that the condition $S^{\mu\nu}P_\nu = 0$ is not adequate for the description of spinning particle motions with the velocity relative Schwarzschild's mass which is greater than some critical value that is close to the velocity of light.

Naturally, the MP equations can be used for description of fermions in the gravitational field only in situations when their quantum properties are not important. Here we stress that the general relativistic Dirac equations is not appropriate for fermions in some highly relativistic regime as well.

It is shown in many papers that in the linear spin approximation the MP equations follow from the general relativistic Dirac equation as some classical approximation (see, e. g. [7]). Here we draw attention to the fact that the exact MP equations (i. e., their nonlinear in spin terms) cannot be obtained from this Dirac equation in principle. Why? To answer this question we recall that the main step in obtaining the general relativistic Dirac equation in the curved spacetime consists in introduction the notion of the parallel transport for spinors as a generalization of this notion for tensors. Whereas according to the MP equations the spin of a test particle is transported by Fermi:

$$\frac{Ds^\mu}{ds} = u^\mu \frac{Du_\nu}{ds} s^\nu, \tag{3}$$

where D/ds is the covariant derivative. It follows from (3) that only in the linear spin approximation the Fermi transport coincides with the parallel transport. Therefore, to satisfy the principle of correspondence between the Dirac equation and the exact MP equations, at first sight, it is necessary simple to introduce and use the Fermi transport for spinors in some corrected Dirac equation. However, it is impossible without the Lorentz invariance violation. In this context we note that many papers are devoted to the violation of Lorentz invariance from different points of view, for example, in the context of the Standard-Model Extension by V. A. Kostelecky and co-authors [8, 9] and, probably, the key words *Lorentz-violating spinor* first appeared only last year [9]. One can hope that just in the framework of this approach the necessary corrected Dirac equation will be obtained.

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