PREHEATING THE UNIVERSE AFTER INFLATION

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ABSTRACT. Inflationary stage is followed by particle production in the background of an oscillating inflaton field, which process is called preheating. For sufficiently strong couplings between the inflaton and matter fields, it is known to proceed non-perturbatively, with parametric resonance playing crucial role for bosonic fields. The evolution of the occupation numbers for fermions is non-perturbative as well. In the Minkowski space, parametric resonance for bosons and non-perturbative effects for fermions would still persist even in the case of weak coupling. In particular, the energy density of created bosons would grow exponentially with time. However, the situation is quite different in the expanding universe. We give a simple demonstration how the conditions of the expanding universe, specifically, redshift of the field modes, lead to the usual perturbative expressions for particle production by an oscillating inflaton in the case of weak couplings. The results that we obtain are relevant and fully applicable to the Starobinsky inflationary model.

Keywords: Inflation; preheating; Starobinsky model

1. Introduction

Consider a scalar field ϕ (to be later associated with the inflaton) of mass M interacting with a light scalar field φ of mass $m_\varphi \ll M$ with the interaction Lagrangian density

$$\mathcal{L}_{\rm int} = -\sigma\phi\varphi^2\,, \tag{1}$$

where σ is a constant of dimension mass. Suppose that the homogeneous field $\phi(t)$ is classically oscillating in the neighborhood of its minimum at $\phi = 0$ with amplitude ϕ_0, while the field φ is in the vacuum state. In the Minkowski space, this initial condition would lead to production of φ-particles via parametric resonance. For sufficiently small values of σ, namely, for

$$\sigma\phi_0 \ll M^2\,, \tag{2}$$

the resonance will be most efficient in the first narrow resonance band centered at the frequency

$$\omega_{\rm res} = \frac{M}{2} \tag{3}$$

(see Shtanov, Traschen and Brandenberger, 1994). Within the resonance band, the mean particle occupation numbers grow with time according to the law

$$N_k = \frac{1}{1-\Delta^2/\sigma^2\phi_0^2}\sinh^2\lambda\, t\,, \tag{4}$$

where

$$\lambda = \frac{1}{M}\sqrt{\sigma^2\phi_0^2-\Delta^2}\,, \qquad \Delta = \omega_k^2-\omega_{\rm res}^2\,, \tag{5}$$

and $\omega_k = \sqrt{m_\varphi^2+k^2} \approx k$ is the frequency of the mode of the field φ. The width of the resonance band is determined by the condition that the expression under the square root in (5) is nonnegative.

The total particle number, as well as the energy density of the φ-particles, in Minkowski space grows asymptotically exponentially with time, in contrast to the expectations based on the naïve perturbation theory, where it grows with time only linearly.

There are several important modifications in the case of expanding universe. Firstly, the amplitude of the oscillating inflaton gradually decreases with time as $\phi_0 \propto a^{-3/2}$, where a is the scale factor. Secondly, the frequency of the mode of the scalar field φ is redshifted:

$$\omega_k = \sqrt{m_\varphi^2+\frac{k^2}{a^2}} \approx \frac{k}{a}\,, \tag{6}$$

where k now is the comoving wave number. Nevertheless, the theory of parametric resonance is still applicable if the evolution of the relevant quantities occurs adiabatically:

$$\left|\dot\phi_0/\phi_0\right| = \frac{3}{2}H \ll M\,, \tag{7}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, and

$$\left|\dot{\lambda}/\lambda\right| \ll \lambda\,, \tag{8}$$

In this case, one can replace law (4) by an approximate expression (Shtanov, Traschen and Brandenberger, 1994)

$$N_k \simeq \sinh^2 \int \lambda\, dt \tag{9}$$

as long as the mode with the comoving wave number k remains within the resonance band.

If the adiabaticity condition (8) does not hold, and the parametric resonance, therefore, does not develop, then one usually employs the Born approximation for the total width Γ_φ of decay of a ϕ-particle into a pair of φ-particles:

$$\Gamma_\varphi = \frac{\sigma^2}{8\pi M}\,. \tag{10}$$

However, if this naïve formula does not work in the Minkowski space (as argued above), one may wonder why it works in the case of expanding universe.

Similar issues can be raised about the production of fermionic particles. Although there is no parametric resonance in this case, still the picture of creation of particle pairs by an oscillating *classical* field is quite different from that based on the usual perturbation theory (Greene and Kofman, 1999, 2000). Nevertheless, in the case of expanding universe, one often uses the Born formula for the total width of decay of ϕ into a pair $\psi, \overline{\psi}$:

$$\Gamma_\psi = \frac{\Upsilon^2 M}{8\pi} \tag{11}$$

where Υ is the Yukawa coupling of the scalar field ϕ to the fermionic field ψ. (The widths (10) and (11) in the Born approximation in the background of an oscillating classical field in the Minkowski space are calculated, e.g., in Shtanov, Traschen and Brandenberger, 1994.)

The purpose of this letter is to clarify the formulated issues and to justify equations (10) and (11) in the case of expanding universe. Our results will be applicable, in particular, to the Starobinsky inflationary model (Starobinsky, 1980), as we will show below.

2. Inflaton decay in the expanding universe

We derive the rates of the inflaton decay into (scalar) bosons and fermions in the expanding universe using the method of Bogolyubov coefficients. For more details, see Rudenok, Shtanov and Vilchinskii, 2014.

A scalar field φ with mass m_φ interacting with the inflaton ϕ via coupling (1) obeys the equation of motion

$$\Box\varphi + \left(m_\varphi^2 + 2\sigma\phi\right)\varphi = 0\,. \tag{12}$$

For the mode $\chi_k = a^{3/2}\varphi_k$ with the comoving wave number k, at the preheating stage, we have the equation (see, e.g., Shtanov, Traschen and Brandenberger, 1994)

$$\ddot{\chi}_k + \Omega_k^2 \chi_k = 0\,, \tag{13}$$

where

$$\Omega_k^2(t) = \omega_k^2(t) + 2\sigma\phi(t) - \frac{9}{4}H^2 - \frac{3}{2}\dot{H}\,, \tag{14}$$

and ω_k is given by (6). At the preheating stage, the inflaton field evolves as

$$\phi(t) = \phi_0(t)\cos Mt\,, \tag{15}$$

where the amplitude $\phi_0(t) \propto a^{-3/2}(t)$ slowly decreases with time due to the universe expansion as a consequence of the adiabaticity condition (7).

Using equation (5), and taking into account that inflaton-field oscillations mainly occur in the regime $\phi_0 \ll M_{\rm P}$, one can see that the adiabaticity condition (8) is violated for weak coupling satisfying (2). It is this case that will be under study in the present paper.

As a consequence of non-stationarity of the external field ϕ and of the metric, the quantity Ω_k in (13) is a function of time. In the case $\Omega_k = \text{const}$, the solution for φ_k would maintain its positive-frequency character, i.e., $\varphi_k \sim e^{{\rm i}\Omega_k t}$ for all t. The time-dependence of Ω_k results in the mixing of frequencies, hence, in particle production of the field φ. In the case under consideration, the resonance band is passed so quickly that parametric resonance does not develop, and particle occupation numbers are small. Therefore, in calculating them, one is justified to use perturbation theory.

The mixing of frequencies is considered in a standard way by looking for solutions of the field equation in the form (Grib, Mamayev and Mostepanenko, 1994)

$$\varphi_k(t) = \frac{1}{\sqrt{\Omega_k}}\Big[\alpha_k(t)e^{{\rm i}\int_{t_0}^t \Omega_k(t')dt'} + \beta_k(t)e^{-{\rm i}\int_{t_0}^t \Omega_k(t')dt'}\Big]\,, \tag{16}$$

$$\dot{\varphi}_k(t) = {\rm i}\sqrt{\Omega_k}\Big[\alpha_k(t)e^{{\rm i}\int_{t_0}^t \Omega_k(t')dt'} - \beta_k(t)e^{-{\rm i}\int_{t_0}^t \Omega_k(t')dt'}\Big]\,, \tag{17}$$

where $\alpha_k(t)$ and $\beta_k(t)$ are the Bogolyubov coefficients satisfying the relation

$$|\alpha_k|^2 - |\beta_k|^2 = 1\,. \tag{18}$$

In terms of these coefficients, the average occupation numbers in the corresponding modes are given by $N_k = |\beta_k|^2$. Substituting expressions (16), (17) into (13), one obtains the system of equations for α_k and β_k,

$$\dot{\alpha}_k = \frac{\dot{\Omega}_k}{2\Omega_k}e^{-2{\rm i}\int_{t_0}^t \Omega_k(t')dt'}\beta_k\,, \tag{19}$$

$$\dot{\beta}_k = \frac{\dot{\Omega}_k}{2\Omega_k}e^{2{\rm i}\int_{t_0}^t \Omega_k(t')dt'}\alpha_k\,, \tag{20}$$

with the initial conditions $\alpha_k = 1$, $\beta_k = 0$. Treating this system perturbatively, in the first order, we replace α_k by unity in (20). Then, leaving only the resonant term in this equation, we use the stationary-phase approximation to evaluate the coefficient β_k (see Rudenok, Shtanov and Vilchinskii, 2014):

$$\beta_k = \frac{\sigma M \phi_0(t_k)}{4\omega_k^2(t_k)} \sqrt{\frac{\pi}{|\dot{\omega}_k(t_k)|}} = \frac{\sigma \phi_0(t_k)}{M^{3/2}} \sqrt{\frac{2\pi}{H(t_k)}} \,, \quad (21)$$

where the moment of time t_k is defined by the stationary-phase relation $\omega_k(t_k) = M/2$, which is just the moment of passing through the center of the resonance band for the k-mode.

The process of particle production can be pictured as follows. A mode with sufficiently high wave number k undergoes redshift till it reaches the resonance region. After passing through the narrow resonance band, it becomes filled with particles with average occupation numbers given by (21), which, in our approximation, remain subsequently constant. The modes with wave number smaller than $k_{\min} = Ma(t_0)/2$, where t_0 is the moment of the beginning of particles creation, will never pass through the resonance region due to the redshift and will be free of particles in our approximation.

In this picture, the energy density $\rho_\varphi(t)$ of the created particles at any moment of time is given by

$$\begin{aligned}\rho_\varphi(t) = \frac{1}{a^4(t)} \int \frac{d^3k}{(2\pi)^3} k |\beta_k|^2 \\ \times \theta\left(k - k_{\min}\right) \theta\left(Ma(t) - 2k\right) \,, \qquad (22)\end{aligned}$$

where $\theta(x)$ is the Heaviside step function.

By comparing the time derivative of this energy density with the equation for the evolution of the energy density ρ_φ of continuously created relativistic particles,

$$\dot{\rho}_\varphi = -4H\rho_\varphi + \Gamma_\varphi \rho_\phi \,, \quad (23)$$

we determine the effective rate of particle production Γ_φ:

$$\Gamma_\varphi \rho_\phi = \left. \frac{k^3 |\beta_k|^2 M \dot{a}}{4\pi^2 a^4} \right|_{k = Ma/2} . \quad (24)$$

Hence, using (21) and relation $\rho_\phi = \frac{1}{2} M^2 \phi_0^2$, we get the standard expression (10) for the quantity Γ_φ.

Consider now the case of fermions. In a curved space-time, one uses the covariant generalization of the Dirac equation:

$$\left[\mathrm{i} \gamma^\mu(x) \mathcal{D}_\mu - m \right] \psi(x) = 0 \,, \quad (25)$$

where $\gamma^\mu(x) = h^\mu_{(a)}(x) \gamma^a$, with γ^a being the usual constant Dirac matrices, and $h^\mu_{(a)}(x)$ is a pseudo-orthonormal tetrad.

We consider the typical case where the spinor field ψ interacts with the inflaton field ϕ through the Yukawa coupling

$$\mathcal{L}_{\mathrm{int}} = \Upsilon \phi \overline{\psi} \psi \,. \quad (26)$$

This leads to the appearance of the effective time-dependent fermion mass in equation (25):

$$m(t) = m_\psi - \Upsilon \phi(t) \,. \quad (27)$$

The Bogolyubov transformation in this case is performed in a usual way as described, e.g., in Grib, Mamayev and Mostepanenko, 1994. The first-order perturbation-theory solution for the Bogolyubov coefficient β_k has the form

$$\beta_k = \frac{\Upsilon M}{4\mathrm{i}} \int_{t_0}^{t} \frac{\phi_0(t')}{\omega_k(t')} e^{2\mathrm{i} \int_{t_0}^{t'} \omega_k(t'') dt'' - \mathrm{i} M t'} dt' \,, \quad (28)$$

where

$$\omega_k = \sqrt{m_\psi^2 + \frac{k^2}{a^2}} \approx \frac{k}{a} \,. \quad (29)$$

Higher-order corrections to this perturbative solution are small under the condition (Shtanov, Traschen and Brandenberger, 1994)

$$\frac{\Upsilon \phi_0}{M} \ll 1 \,, \quad (30)$$

which is assumed to be the case.

By using the stationary-phase approximation, we obtain, similarly to (21) (see Rudenok, Shtanov and Vilchinskii, 2014),

$$\begin{aligned}\beta_k &= -\frac{\Upsilon M \phi_0(t_k)}{4\omega(t_k)} \sqrt{\frac{\pi}{|\dot{\omega}_k(t_k)|}} \\ &= -\frac{1}{2} \Upsilon \phi_0(t_k) \sqrt{\frac{2\pi}{M H(t_k)}} \,. \qquad (31)\end{aligned}$$

Then, repeating the reasoning of the case of bosons, and taking into account the four spin polarizations of particles and anti-particles, we get the final result for the production rate of fermions Γ_ψ, which coincides with (11).

3. The Starobinsky model

Historically, one of the first inflationary models was the model suggested by Starobinsky in 1980. It is motivated by the necessity to consider local quantum corrections to the Einstein theory of gravity. The simplest such correction represents the term proportional to the second power of the Ricci scalar in the action of the model, so that the full gravitational action reads as

$$S_{\mathrm{g}} = -\frac{M_{\mathrm{P}}^2}{2} \int d^4x \sqrt{-g} \left(R - \frac{R^2}{6\mu^2} \right) \,, \quad (32)$$

where

$$\mu = 1.3 \times 10^{-5} M_{\mathrm{P}} \quad (33)$$

is a constant with indicated value required to explain the inflationary origin of the primordial perturbations (Faulkner et al., 2006).

A conformal transformation $g_{\mu\nu} \to \chi^{-1} g_{\mu\nu}$, with $\chi = \exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}\right)$ transforms the theory (32) into the usual Einstein gravity with a new special scalar field ϕ (the scalaron):

$$S_{\rm g} = -\frac{M_{\rm P}^2}{2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \quad (34)$$

where

$$V(\phi) = \frac{3\mu^2 M_{\rm P}^2}{4}\left[1-\chi^{-1}(\phi)\right]^2 \quad (35)$$

is the arising field potential.

The scalaron universally interacts with other fields present in the theory. At the preheating stage, we can employ the relation

$$|\phi/M_{\rm P}| \ll 1\,. \quad (36)$$

Without taking into account the back-reaction of the mater fields on the dynamics of the scalaron field, its behavior at the stage of preheating is approximately described by the Klein–Gordon equation

$$\Box\phi + \mu^2\phi = 0\,, \quad (37)$$

and by the oscillatory regime (15) with mass $M = \mu$.

Due to condition (36) and assuming also that $m_\varphi \ll \mu$, one can obtain an approximate equation for a scalar field φ minimally coupled to gravity in the original frame (32):

$$\Box\varphi + \left[m_\varphi^2 - \frac{\mu^2\phi}{\sqrt{6}M_{\rm P}}\right]\varphi = 0\,. \quad (38)$$

This is just equation (12) with

$$\sigma = -\frac{\mu^2}{2\sqrt{6}M_{\rm P}}\,, \quad (39)$$

Thus, the theory of Sec. 2 is applicable here, and the particle production rate is given by (10):

$$\Gamma_\varphi = \frac{\sigma^2}{8\pi M} = \frac{\mu^3}{192\pi M_{\rm P}^2}\,, \quad (40)$$

which coincides with equation (7) of Gorbunov and Panin, 2010.

For the fermions, we derive the decay rate (11):

$$\Gamma_\psi = \frac{\Upsilon^2 M}{8\pi} = \frac{\mu m_\psi^2}{12\pi M_{\rm P}^2}\,, \quad (41)$$

which, up to a factor four (possible account of the spin states), coincides with equation (8) of Gorbunov and Panin, 2010.

4. Discussion

Particle production in the background of an external classical oscillating field is one of the key processes describing the stage of preheating after inflation. In Minkowski space, it would be dominated by the parametric resonance in the lowest resonance band, no matter how small is the coupling between the inflaton and bosonic matter fields. The energy density of the created particles would grow exponentially with time, in contrast to the usual perturbation-theory expectations.

The specific features of the expanding universe, surprisingly, restore the validity of the usual Born formula in the case of sufficiently weak coupling. The reason is that every particular mode of the field to be excited, due to redshift, quickly passes the resonance zone, resulting in small occupation numbers.

It is also important that couplings (1) and (26) were linear in the inflaton field ϕ, so that we also had $N_k = |\beta_k|^2 \propto \phi_0^2 \propto \rho_\phi$. These are precisely the types of coupling responsible for the decay of a ϕ-particle, justifying the interpretation of (10) and (11) as decay rates.

We have shown that the results of the present paper are fully applicable to one of the most successful inflationary models — the Starobinsky model.

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References

Linde A.D.: 1990, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland).

Traschen J.H., Brandenberger R.H.: 1990, *Phys. Rev. D*, **42**, 2491.

Kofman L., Linde A.D., Starobinsky A.A.: 1994, *Phys. Rev. Lett.* **73**, 3195.

Shtanov Y., Traschen J.H. and Brandenberger R.H.: 1995, *Phys. Rev. D*, **51**, 5438.

Greene P.B., Kofman L.: 1999, *Phys. Lett. B*, **448**, 6.

Greene P.B., Kofman L.: 2000, *Phys. Rev. D*, **62**, 123516.

Starobinsky A.A.: 1980, *Phys. Lett. B*, **91**, 99.

Grib A.A., Mamayev S.G., Mostepanenko V.M.: 1994, *Vacuum Quantum Effects in Strong Fields* (Friedmann Laboratory Publishing, St.-Petersburg).

Faulkner T. et al.: 2006, *Phys. Rev. D*, **76**, 063505.

Gorbunov D.S., Panin A.G.: 2011, *Phys. Lett. B,* **700**, 157.

Rudenok I., Shtanov Y., Vilchinskii S.: 2014, *Phys. Lett. B,* **733**, 193.