THE ESTIMATION OF THE HEIGHT AND HALF-WIDTH OF THE TWILIGHT BEAM IN EARTH ATMOSPHERE

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ABSTRACT. New program models the scattering of the sun light in Earth atmosphere. The program is not in the strict relation with the model of light scattering and atmosphere structure. This fact permits to use the program for different atmosphere models. The way of determination of the twilight beam characteristics is discussed in detail. The article presents the results of the model for average atmosphere. They may be used for determine of the direction scattering coefficient and execute iterations. The given is model similar to the average model used in N.B. Divari' articles. This fact caused the similarity of the obtained results.

Key words: twilight, atmosphere, twilight sound, twilight beam, optic depth of the atmosphere.

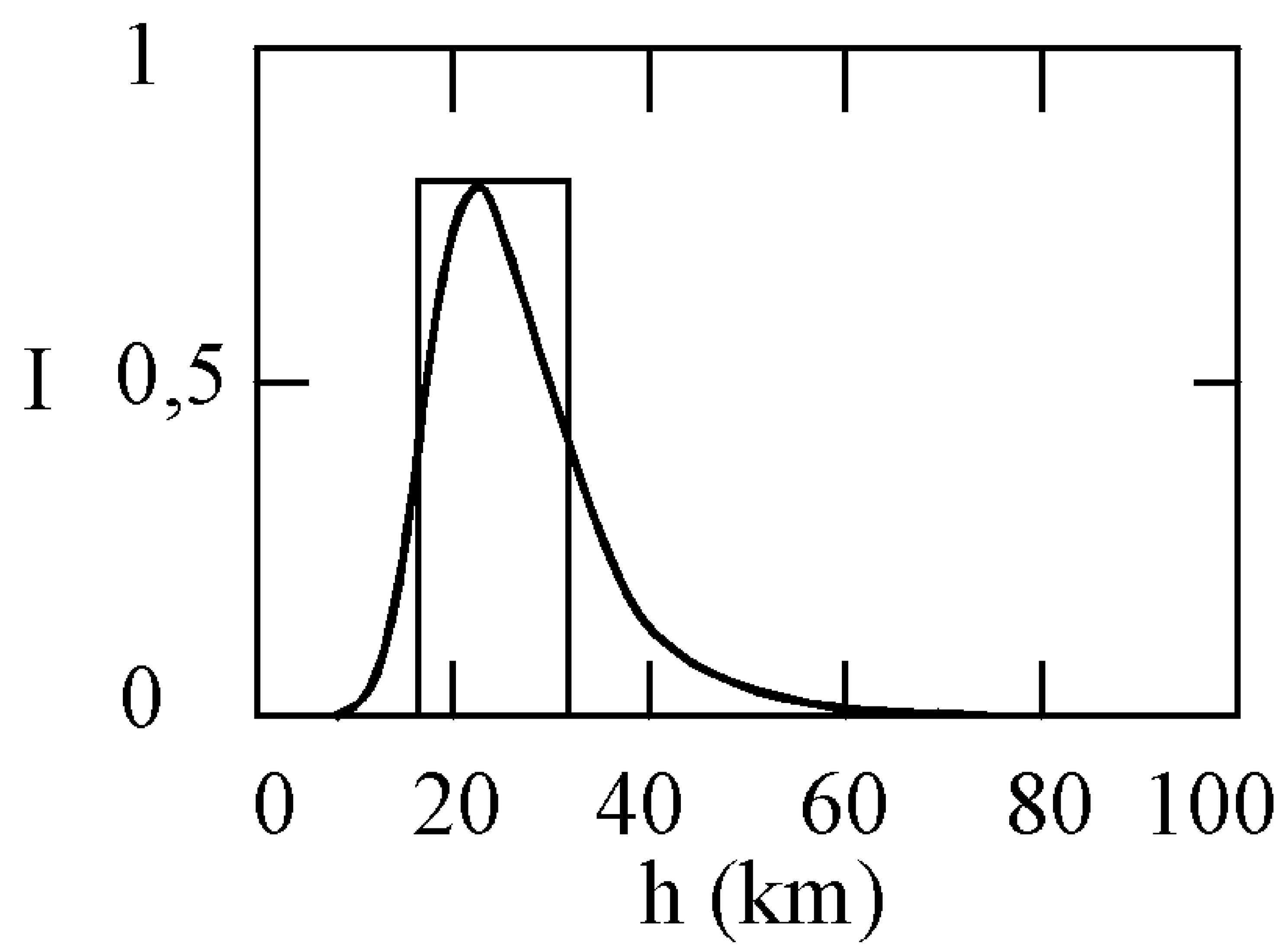

Figure 1:

One of the methods to determine Earth atmosphere dust is the method of twilight sounding, which it basic on solving of the inverse problem of the light scattering in the atmosphere. The result of solving the inverse problem is the directed scattering coefficient which is determined out of the visible brightness of the twilight sky.

Primary twilight brightness may be present as

$$B_1 = B_m + B_d + B_o, \qquad (1)$$

where B_m - the brightness of primary twilight, caused by molecular component,
B_d - the brightness of primary twilight, caused by dust component,
B_o - the brightness of primary twilight, caused by ozone component.

Each component causes brightness, which may be obtained after the formula

$$B = \beta\psi(\theta) \int_0^\infty I_0 n(h) e^{-\tau} e^{-\tau'} dh, \qquad (2)$$

where B - the brightness, caused by one of the components,
$e^{-\tau}$ - decreasing of a light flaw up to a scattering point,
$e^{-\tau'}$ - decreasing of a light flaw from a scattering point up to observer,
β - the scattering coefficient of an appropriate component,
I_0 - the intensity of solar radiation outside the atmosphere at the distance of the Earth orbit,
$n(h)$ - the concentration of particles of appropriate component at the height h,
$\psi(\theta)$ - the indicatrix function of appropriate component.

N.B. Divari (Divari 1972,1973,1974) offers to substitute profile of the twilight beam by rectangular (fig. 1),
where I_h - solar radiation intensity at height h,
h - scattering point height.

The height of the rectangle equals the maximum of the integrand (2) and its width is difference of heights, where the value of integrand is two times less than maximum value (5).

$$\frac{dB}{dh} = \beta\psi(\theta) I_0 n(h) e^{-\tau} e^{-\tau'}, \qquad (3)$$

$$\frac{dB}{dh}(h') = max\left(\frac{dB}{dh}(h)\right), \qquad (4)$$

$$\Delta h = h_2 - h_1, \qquad (5)$$

where h'- the height of the twilight beam,
Δh - the half-width of the twilight beam,
$h_{1,2}$ - the height, where brightness of the scattering light is two times less than maximum brightness of the scattering light (6).

$$\frac{dB}{dh}(h_{1,2}) = \frac{1}{2} max\left(\frac{dB}{dh}(h)\right). \qquad (6)$$

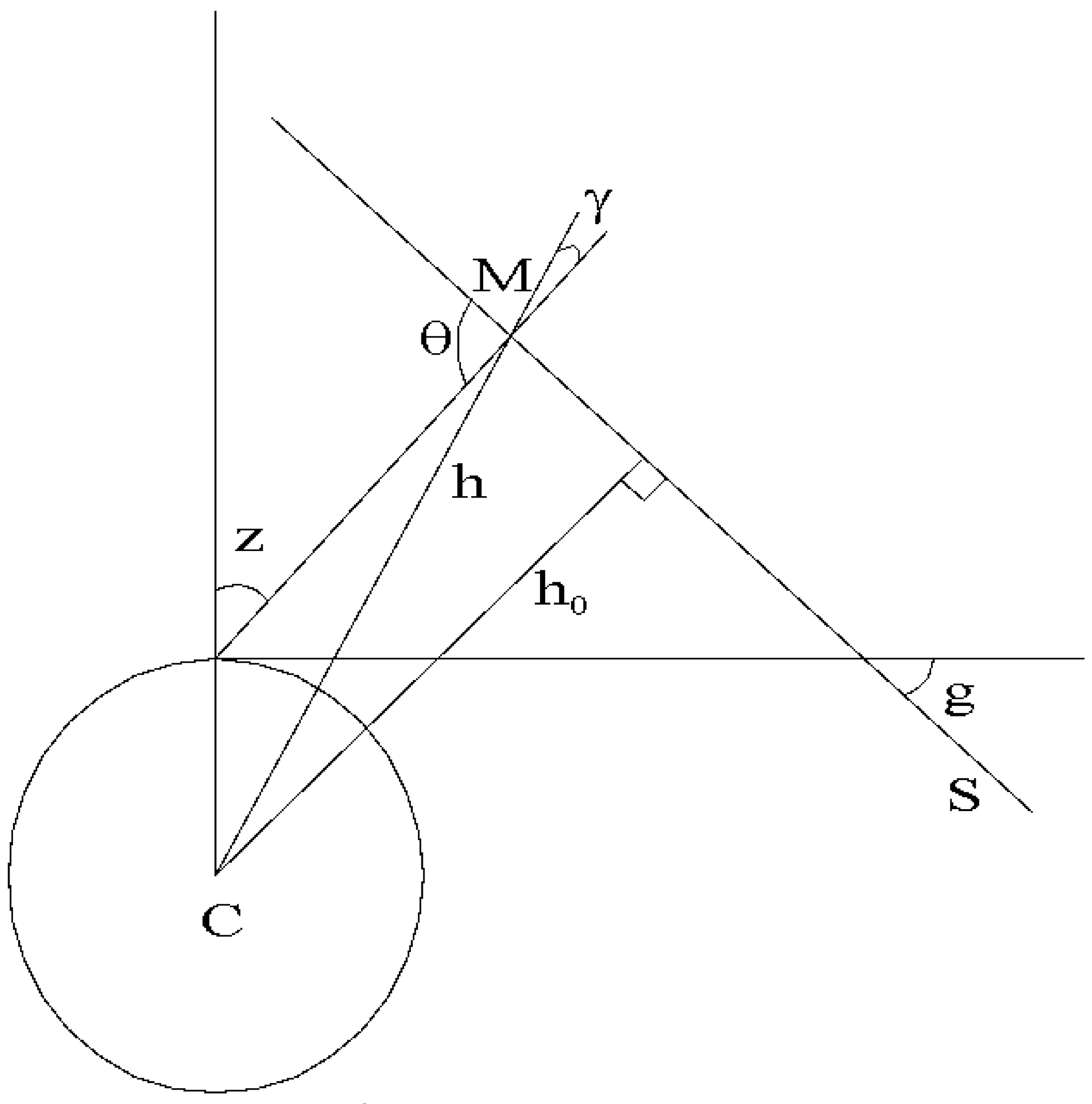

Figure 2:

As a result this substitution transforms integrate equation (2) in algebraic equation (7).

$$B = I_0 \sigma(\theta, h') e^{-(\tau+\tau')} sec(\gamma) \Delta h, \qquad (7)$$

where $\sigma(\theta, h')$ - the directed scattering coefficient.

Thus, the solving of the inverse problem needs to knowing angels and , height of the twilight beam, its half-width and optic depth along twilight beam. Angels and may be easily determined out of geometrical ideas (fig. 2). The height of the twilight beam, its half-width and optic depth may be determined only using the results of modelling the light scattering in atmosphere. As follows, the result of solving the inverse problem will depended from the choice of the prior atmosphere model. It may be assumed, that solving the inverse problem will produce the atmosphere model more similar to reality than prior model. Lets use iteration method.

The task was formulated to obtain the height, half-width of the beam and optic depth along the twilight beam for every observing day by iteration. Within the discussed problem we made program which calculates the profile of the twilight beam for any prior atmosphere. To determine the twilight beam profile. Its needed to calculate optic depth along any trajectory. Optic depth in atmosphere may be present as

$$\tau = \tau_m + \tau_d + \tau_o, \qquad (8)$$

where τ_m - the optical depth of the atmosphere molecular component,
τ_d - the optical depth of the atmosphere dust component,
τ_o - the optical depth of the atmosphere ozone component.

The optic depth of any component may be calculated after the formula (9) (Divari and Plotnicova 1965; Lebedinech 1981)

$$\tau = \int_{h_0}^{\infty} \alpha n(h) \frac{R+h}{\sqrt{(R+h)^2 - (R+h_0)^2}} dh, \qquad (9)$$

where R - the Earth radius,
h_0 - the minimum distance of the beam from the Earth surface,
α - the extinction factor.

Though the given formula is simple enough for analytic it is principal unsuitable for calculation near $h = h_0$. When $h = h_0$ the integrand becomes to infinite. This may be avoided by integration along the trajectory. The transformation Jacobean excluded from formula (9).

$$\frac{\partial S}{\partial h} = \frac{R+h}{\sqrt{(R+h)^2 - (R+h_0)^2}} \qquad (10)$$

and we get the formula, where optic depth is

$$\tau = \alpha \int_0^{\infty} n(h) ds, \qquad (11)$$

where α- the factor of absorption for molecules and ozone and the dispersion factor for dust,
$n(h)$ - the concentration of particles of appropriate component at height h,
ds - the beam trajectory element.

To estimate the quality of the integration we may use a value estimation of the remainder. For quadrature integrate formulas the remainder member may be presented as follows (12) (Demidovich and Maron 1963)

$$R = Al^n f^{(n)}(\xi), \qquad (12)$$

where $f^{(n)}(\xi)$ - n rank derivative from integrate function at point the , that belong to the interval of integration,
l - step of integration,
A - constant.

The calculation offer formula (11) is more effective, because the $n-$ rank derivative of integrand function from formula (11) near $h = h_0$ is less than integrand from formula (9).

Knowing optic depth along the trajectory of the beam, we can determinate the profile of the twilight beam (3). Using the profile of the twilight beam we can determinate the height of the beam h and it half-width h in above-mentioned way.

In formula (1) we assumed that model of atmosphere includes molecular, ozone and dust components. The absorption and scattering by molecular and ozone components can be determined if absorption and scattering coefficients of those two components is known and the dependence of molecular concentration from height and molecular indicatrix. The question which concerns the dust component is more complicated because it consists from the particles of different size, that scatter radiation indifferent way evidently. The scattering coefficient cant be equivalent for all particles. Therefore to simplify we refused from atmosphere model that contains particles different size. We suppose that instead the atmosphere has "effective" particles with identical

Table 1:

λ (nm)	$\beta_m(cm^{-1})$	$\beta_d(cm^{-1})$	$\beta_o(cm^2)$	$\alpha_m(cm^{-1})$	$\alpha_o(cm^2)$
371	$4.83 \cdot 10^{-8}$	$0.13 \cdot 10^{-8}$	0	$0.607 \cdot 10^{-6}$	0
575	$7.80 \cdot 10^{-9}$	$0.13 \cdot 10^{-8}$	$0.0382 \cdot 10^{-25}$	$0.955 \cdot 10^{-7}$	$0.0471 \cdot 10^{-19}$

Table 2:

h(km)	lgN (cm^{-3})	h	lgN	h	lgN
0	19.41	20	18.27	120	11.69
1	19.36	30	17.58	150	10.66
2	19.31	40	16.92	200	9.86
3	19.28	50	16.34	250	9.3
4	19.23	60	15.82	300	8.9
5	19.19	70	15.26	400	8.1
6	19.14	80	14.60	500	7.4
8	19.04	90	13.80	700	6.4
10	18.98	100	12.98	1000	5.2
15	18.61	110	12.29		

Table 3:

h(km)	n(cm^{-3})	h(km)	n(cm^{-3})
10	$1.0 \cdot 10^{12}$	60	$1.0 \cdot 10^{10}$
15	$1.1 \cdot 10^{12}$	65	$3.2 \cdot 10^{9}$
20	$2.9 \cdot 10^{12}$	70	$1.0 \cdot 10^{9}$
25	$3.2 \cdot 10^{12}$	75	$3.2 \cdot 10^{8}$
35	$2.0 \cdot 10^{12}$	80	$1.4 \cdot 10^{8}$
40	$1.0 \cdot 10^{12}$	85	$1.0 \cdot 10^{8}$
45	$3.2 \cdot 10^{11}$	90	$1.1 \cdot 10^{8}$
50	$1.0 \cdot 10^{11}$	95	$1.3 \cdot 10^{7}$
55	$3.2 \cdot 10^{10}$	100	$1.7 \cdot 10^{6}$

scattering coefficient and indicatrix. The concentration of these particles is such, that they scatter light equally to its scattering by all particles, of atmosphere. Then dust component of the atmosphere can be set by the scattering coefficient of the "effective" particles and they distribution by height.

$$n(h) = \begin{cases} 7.1e^{-0.1h}, & h \leq 40km \\ 4.8 \cdot 10^{10} h^{-7.2}, & h > 40km \end{cases} \quad (13)$$

Factors of scattering and absorption a presented in table 1 (Divari and Plotnicova 1965; Sedunov 1991), the concentration of a molecular in table 2 (Allen 1977), concentration of ozone in table 3 (Sedunov 1991).

The column herders used in the table 1 are
β - scattering factors,
α - absorption factors.

For molecular and dust component the factors are the decrising of light flux per length unit at sea level and for ozone the decrising at crossing volume unit.

Scattering indicatrix of molecular and dust according to (Divari and Plotnicova 1965) a represented by formulas (14,15).

$$\psi(\theta) = 3.046 \cdot 10^{-4} \frac{3}{16\pi}(1 + cos^2\theta) \quad (14)$$

$$\psi(\theta) = \frac{1}{18.28}[1 + 11.1(e^{-3\theta} - 0.009)] \quad (15)$$

Using the given atmospheric model we obtained heights and half- widths twilight beams as well optic depths along them (tab. 4,5). Calculating the characteristics of twilight beam we supposed, that beams in atmosphere have linear of trajectories and Sun is a point source. Thus we neglect with effects of refraction, refraction divergence and limited size of the Sun. These effects change a little bit the numeric values of twilight beam characteristics but dont change qualitative dependence of the twilight beam the height and its half-width on direction of the observation and sun immersion.

The designations used in table 4, 5 are
g - Sun immersion under horizon,
z - zenith distance of observation point. The negative values of z are zenith distances merged from zenith point in the direction to the opposite Sun. τ - optic depth from Sun to observer,
h' - twilight beam height,
Δh - its half-width,
λ - wave length.

On the basis of date given in table 4, 5 and formula (2) we may obtaining the scattering direction factor if the brightness of primary twilight is known. The scattering direction factor may be used for repeated obtain of the twilight beam characteristics. In order to do this atmosphere is to be interpreted as consisting of the only component and scattering direction factor as a product of concentration of particles of mentioned component and indicatrix same component.

As all absorption and scattering factors, molecular and dust indicatrix and concentration of the "effective" particles were got from N.B. Divari works we compared heights of twilight beams obtaining by us and by N.B. Divari (fig. 3).

On figure 3 we put regular bias in 5km for heights obtained by N.B. Divari in order to make curves not to coincide.

hm - twilight beam heights obtained by us,

Table 4:

λ =371(nm) gas,dust				
g	z	lg($e^{-\tau}$)	h' (km)	Δ h (km)
2	80	-1,594	24,65	14,77
4	80	-1,612	32,03	14,65
6	80	-1,548	43,56	14,00
10	80	-1,512	74,05	15,38
14	80	-1,400	117,24	22,35
2	70	-1,057	25,84	16,19
4	70	-0,978	36,51	15,96
6	70	-0,947	50,49	16,41
10	70	-0,909	90,37	18,86
14	70	-0,762	148,06	28,41
2	60	-0,813	27,42	16,28
4	60	-0,777	38,36	15,15
6	60	-0,742	54,18	16,26
10	60	-0,667	100,88	22,64
14	60	-0,556	167,39	31,75
2	50	-0,723	27,83	16,39
4	50	-0,678	39,52	15,88
6	50	-0,690	55,65	15,48
10	50	-0,538	108,49	24,97
14	50	-0,457	181,61	34,91
2	40	-0,675	28,12	16,41
4	40	-0,625	40,35	16,70
6	40	-0,633	57,38	16,19
10	40	-0,482	113,73	25,54
14	40	-0,400	193,09	36,48
2	30	-0,645	28,35	16,44
4	30	-0,647	40,05	15,42
14	30	-0,366	203,06	38,15
2	20	-0,629	28,54	16,42
4	20	-0,627	40,60	15,66
6	20	-0,579	59,99	19,24
10	20	-0,402	123,04	27,13
14	20	-0,344	212,34	39,81
2	10	-0,621	28,72	16,46
4	10	-0,681	40,13	15,98
6	10	-0,568	61,12	17,78
10	10	-0,391	126,87	27,95
14	10	-0,320	222,48	41,28
2	-80	-1,711	39,29	16,90
4	-80	-1,889	74,22	26,82
6	-80	-1,732	203,73	81,08
2	-70	-1,022	33,72	18,30
4	-70	-1,191	50,45	18,64
6	-70	-1,053	90,43	29,75
10	-70	-0,869	285,18	72,65
2	-60	-0,807	32,06	18,36
4	-60	-0,906	46,72	18,71

Table 4 (Continue)

λ =371(nm) gas,dust				
g	z	lg($e^{-\tau}$)	h' (km)	Δ h (km)
6	-60	-0,847	76,17	20,80
10	-60	-0,613	196,38	46,23
2	-50	-0,702	31,31	17,47
4	-50	-0,804	44,53	17,76
6	-50	-0,746	70,55	19,81
10	-50	-0,513	168,55	38,15
2	-40	-0,700	29,82	19,11
4	-40	-0,750	43,25	17,17
6	-40	-0,691	67,41	18,63
10	-40	-0,459	154,76	34,26
2	-30	-0,661	29,52	18,84
4	-30	-0,715	42,37	16,80
6	-30	-0,651	65,30	18,07
10	-30	-0,423	146,14	32,19
2	-20	-0,639	29,28	18,56
4	-20	-0,693	41,70	16,57
6	-20	-0,580	64,76	19,38
10	-20	-0,402	139,94	30,73
2	-10	-0,626	29,08	18,35
4	-10	-0,683	41,13	16,35
6	-10	-0,569	63,43	18,78
10	-10	-0,391	135,02	29,73

Table 5:

λ =575(nm) gas,dust,ozone				
g	z	lg($e^{-\tau}$)	h' (km)	Δ h (km)
2	80	-0,887	14,60	16,39
4	80	-1,043	22,32	19,31
6	80	-1,158	33,92	21,44
10	80	-1,213	64,81	30,07
14	80	-0,387	122,28	28,41
2	70	-0,774	16,48	16,97
4	70	-0,977	25,19	21,76
6	70	-0,986	39,01	11,38
10	70	-1,071	79,89	39,53
14	70	-0,229	153,02	32,48
2	60	-0,807	15,99	17,35
4	60	-0,954	26,56	21,93
6	60	-1,009	41,14	10,36
10	60	-0,990	89,52	44,51
14	60	-0,166	173,57	36,45
2	50	-0,799	16,25	17,54
4	50	-0,945	27,40	21,34
6	50	-1,068	42,10	12,75

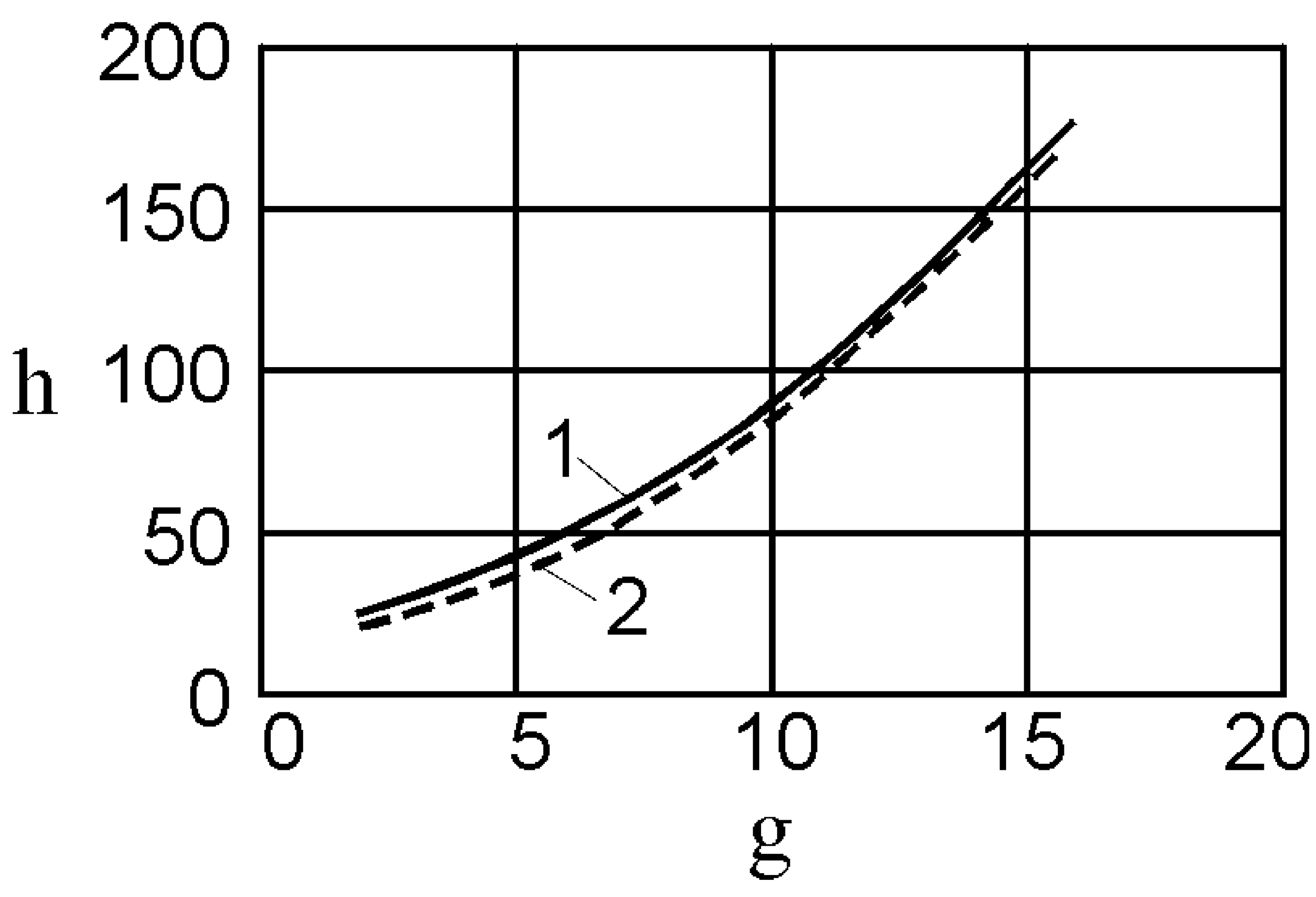

Figure 3:

hd - twilight beam heights after by N.B. Divari,
g - Sun immersion under horizon.

You can easily notice that our results are very similar to those obtained by N.B. Divari through all the heights range.

Thus as a result we have obtained program that models twilight scattering for any atmosphere if supposition of linear spreading of sunbeams and neglecting limited size Solar dimensions. The values that characteristics the twilight beams for prior given atmosphere are obtained. The comparison of twilight beam heights shows they identity, that witness the correctness of program running and although the absence of strong correlation of twilight beam height with the prior selected molecular component.

References

Divari N.B.: 1973, *Pyl v atmosfere i okolozemnom prostranstve*, Nauka, Moscow, 75.

Divari N.B., Plotnikova L.I.: 1965, *Astron. Zhurn.*, **42**, 1090.

Lebedenec V.N.: 1981, *Ajerosol v vertchnej atmosfere i kosmicheskaja pyl*, Gidrometeoizdat, Lenigrad, 16.

Allen K.U.: 1977, *Astrofizicheskie velichiny*, Mir.

Sedunov Ju.S.: 1991, *Atmosfera*, Gidrometeoizdat, Leningrad, 287.

Divari N.B.: 1970, *Atmosfernaja optika*, Nauka, Moscow, 158.

Demidovich B.P., Maron I.A.: 1963 *Osnovy vychislitelnoj matematiki*, Fizmatgiz, Moscow, 604.

Divari N.B.: 1967, *Kosmicheskie issledovania*, **5**, Moscow, 475.

Divari N.B.: 1972, *Bull. Abastuman. Astrophys. Obs. observatorii*, **41**, 81.

Divari N.B.: 1974, *Atmosfernaia optika*, Nauka, Moscow, 145.

Table 5 (Continue)

λ =575(nm) gas,dust,ozone				
g	z	lg($e^{-\tau}$)	h' (km)	Δ h (km)
10	50	-0,993	94,77	49,35
14	50	-0,142	188,21	38,58
2	40	-0,742	17,40	17,68
4	40	-0,937	27,99	20,46
6	40	-1,029	45,30	21,53
10	40	-0,894	102,10	50,88
14	40	-0,128	200,04	40,50
2	30	-0,739	17,55	17,74
4	30	-0,937	28,46	21,12
14	30	-0,120	210,35	42,31
2	20	-0,740	17,67	17,74
4	20	-0,936	28,87	21,79
6	20	-1,017	47,38	22,22
10	20	-0,183	129,74	45,13
14	20	-0,115	219,93	45,94
2	10	-0,741	17,78	19,76
4	10	-0,938	29,23	20,43
6	10	-1,016	48,28	22,62
10	10	-0,179	133,77	46,59
14	10	-0,112	229,40	47,83
2	-80	-1,163	20,80	20,97
4	-80	-1,989	40,92	23,50
6	-80	-1,329	154,38	62,21
2	-70	-0,923	18,89	19,59
4	-70	-0,930	38,71	12,23
6	-70	-1,187	70,41	28,40
10	-70	-1,039	250,34	127,91
2	-60	-0,866	18,05	19,49
4	-60	-1,027	33,94	20,40
6	-60	-1,071	60,96	26,86
10	-60	-0,961	172,70	89,76
2	-50	-0,838	17,66	19,19
4	-50	-0,992	32,40	20,03
6	-50	-1,040	56,56	24,56
10	-50	-0,937	148,62	76,10
2	-40	-0,770	18,45	18,76
4	-40	-0,973	31,49	20,54
6	-40	-1,032	54,09	24,56
10	-40	-0,197	163,24	57,30
2	-30	-0,765	17,25	16,90
4	-30	-0,960	30,86	19,36
6	-30	-1,024	52,42	23,31
10	-30	-0,915	129,07	66,44
14	-30	-0,120	281,56	56,71
2	-20	-0,749	18,13	18,41
4	-20	-0,952	30,37	20,56
6	-20	-1,019	51,15	24,44
10	-20	-0,184	147,56	51,33
2	-10	-0,745	18,00	18,14
4	-10	-0,945	29,96	19,80
6	-10	-1,017	50,10	23,66
10	-10	-0,181	142,36	49,47